

# Modelling port competition from a transport chain perspective

Dong-Ping SONG, Andrew LYONS, Dong LI, Hossein SHARIFI

(accepted version)

*School of Management, University of Liverpool, Chatham Street, Liverpool, L69 7ZH, UK,  
Email: [dongping.song@liverpool.ac.uk](mailto:dongping.song@liverpool.ac.uk); [etmalyon@liverpool.ac.uk](mailto:etmalyon@liverpool.ac.uk); [dongli@liverpool.ac.uk](mailto:dongli@liverpool.ac.uk);  
[sharifi@liverpool.ac.uk](mailto:sharifi@liverpool.ac.uk);*

## Abstract

This paper considers the competition between two ports involving both hinterland shipments and transshipments. Taking a transport chain perspective including deep-sea, port, feeder and inland transportation, we present a static cost model to examine ports' relative competitiveness and justify the development of game models. A non-cooperative game model is then formulated for a two-ports-one-ocean carrier system. The optimal ports' pricing and the carrier's port-of-call decisions are derived. A centralized supply chain model is then discussed. The game model is further extended to uncertain demand situations. A case study of Southampton and Liverpool ports is provided to illustrate the results.

**Keywords:** port competition; container shipping; transport chain; game theory; uncertainty.

## 1. Introduction

Port competition is an accepted and important phenomenon, and a key driver of performance improvement, in the shipping industry. This is particularly evident in the container shipping sector where container port operations, cargo handling and equipment are standardized. Competition is intensified as ocean carriers can relatively easily switch their service routes and ports of call (denoted as portcall for simplicity) between different container ports. For example, among the UK container ports, in recent years it was reported that Evergreen moved to Felixstowe from Thamesport; a joint Hapag-Lloyd/OOCL transatlantic service was switched to Southampton from Thamesport; BG Freight Line (a subsidiary of CMA CGM) moved most of its services from Tilbury to Thamesport; the Southern Africa Europe Container Service was switched to London Gateway port from Tilbury (Porter 2013).

Many factors affect ocean carriers' and shippers' decisions on the selection of ports, e.g. availability of hinterland connections, port tariffs, immediacy of consumers (large hinterland), feeder connectivity, environmental issues and the total portfolio of the port (Wiegman et al., 2008). From a global supply chain perspective, the total transport chain's cost/ profit is regarded as the most significant criterion for port choice (Liu et al. 2014). This paper attempts to address the competitive challenge between two container ports involving both hinterland shipments and transshipments from the transport chain's cost perspective including port prices, deep sea transport cost, hinterland transport cost, and feeder service cost.

There is a rich and varied body of literature on the subject of port competition. Port competition may be classified into three categories: intra-port competition between terminal operators within a single container port, inter-port competition between operators/ authorities in neighbouring ports, and inter-port competition between operators/ authorities in different port ranges. A typical example of the first category is the rivalry among the three major terminals in Rotterdam: the Euromax Container Terminal (operated by Hutchison Ports), the Rotterdam World Gateway terminal (operated by DP World), and the APMT MVII terminal (operated by APM Terminals) (Barnard 2014). Another example has been highlighted by

Saeed and Larsen (2010) who studied the intra-port competition among three container terminals located in a port in Pakistan, and examined the different types of coalitions among the container terminals using a two-stage game method.

In the second category, competitive ports are located in the same port region competing for the same hinterland shipments (and may also compete for the same transshipments). For example, Southampton and Liverpool ports compete for the hinterland shipments from England and also compete for the transshipment cargoes from Scotland and Ireland. Cullinane et al. (2005) analyzed the relative competitiveness of the two neighbouring container ports of Shanghai and Ningbo with respect to price, quality of service and generalized cost. De Borger et al. (2008) applied a two-stage game to analyze the interaction between the pricing behaviour of two competing ports and the capacity investment policies in the ports and hinterland. Both port congestion and hinterland congestion are considered in the model. Li and Oh (2010) studied the competition and cooperation between neighbouring ports in a case study of Shanghai port and Ningbo-Zhoushan port. Luo et al. (2012) developed a two-stage game model for a new port and an existing port that serve the same hinterland with different competitive conditions. They focused on port pricing and capacity expansion decisions. The case of Hong Kong and Shenzhen ports was discussed.

In the third category, competitive ports are located in different port ranges and therefore mainly compete for transshipment cargoes. Veldman and Buckmann (2003) applied a logit model to quantify the routing choice among European container hub-ports. Yap and Lam (2006) examined whether there exists a long run relationship between various ports in East Asia using a co-integration test based on historical data. Co-integration refers to a linear combination of variables that are non-stationary with a relationship present between them. Anderson et al. (2008) investigated the competition between two hub ports: Busan and Shanghai. They developed a game-theoretic response model for the purpose of understanding how a competing port would best respond to the development of the focal port, and whether the focal port would be able to capture or defend market share through investment in capacity. Ishii et al. (2013) applied a non-cooperative game theoretic model to examine the effect of inter-port competition between two ports using the case of Busan and Kobe. Working under the assumption that both the levels and timings of capacity investment are pre-determined, they aimed to determine the pricing behaviour of the two ports at each time period of port capacity investment. Zhuang et al. (2014) used duopoly games to model the competition between two ports that service two types of cargoes. They found that inter-port competition may lead to port specialization in terms of port service choice and cargo type. Bae et al. (2013) studied container port competition for transshipment cargoes in a duopoly market. A non-cooperative game was applied to a vertical marketing channel consisting of two ports and multiple shipping lines. They showed the existence of the Nash equilibrium including shipping lines' portcall decisions and ports' pricing decisions. A defining contribution of this paper is the joint/ interactive decision-making of ports and shipping lines, while most other literature on port competition has primarily focused on the ports' decisions only.

In addition to port competition, there have been a number of empirical studies examining the competitiveness of container ports. For example, Tongzon and Heng (2005) conducted an empirical evaluation of the impact of port privatization on port efficiency and identified the determinants of port competitiveness. Yeo et al. (2008) considered the competitiveness of container ports in the regions of Korea and China. They conducted a regional survey of shipping companies to identify and evaluate the determining factors influencing port

competitiveness. Notteboom and Yap (2012) discussed port competition and competitiveness. They introduced the concept of container ‘port range’, which is defined as a geographically defined area with a number of ports that possess largely overlapping hinterlands and thus serve mostly the same customers. Related to port competition and competitiveness, other researchers have addressed the issues of port cooperation and regionalization. For example, Song (2002) took a strategic perspective to examine the possible competition and cooperation between Hong Kong port and the adjacent container ports in South China. It was reported that port cooperation could be achieved through the same terminal operator or through common ownership. Luo and Grigalunas (2003) presented a simulation model to estimate port-related demand for major US coastal container ports. The demand regionalization was achieved through simulating the multimodal container transportation process based on the shortest path method.

It can be observed that the literature on port competition has focused on either competing for hinterland shipments, or competing for transshipments. Very little research has considered port competition involving both hinterland shipments and transshipments explicitly. Given the fact that the majority of deep sea ports handle both hinterland shipments and transshipment cargoes (although their ratios may vary from port to port), it is appropriate to model port competition by including both types of shipments. More importantly, the port competition models developed so far have primarily concentrated on the port performance and related decisions (e.g. price, investment, congestion); ocean carriers’ decisions have been often neglected except in one paper (i.e., Bae et al. 2013). Since ocean carriers are the immediate and primary customers of container ports, and ocean carriers’ portcall decisions depend on the entire transport chain, it is desirable to model port competition in the context of the transport chain by considering port pricing, deep sea transport cost, hinterland transport cost, and feeder service cost simultaneously. In addition, it is also useful to investigate the centralized management model for the transport chain in an integrated manner since ports and ocean carriers may seek strategic collaboration and make decisions jointly. A loosely related research stream is shipping network design (Brouer et al. 2014; Meng et al. 2014), which is aimed at designing or selecting shipping service routes, port choice, port rotation, and inland transportation in order to meet customer demands (Tavasszy et al. 2011; Liu et al. 2014). However, this research stream (on shipping network design) has not considered the competition between ports, i.e. port pricing has not been treated as a decision variable. In this study, we aim to address the port competition and ocean carrier’s port-of-call decision problem from the transport chain’s perspective by considering deep sea, port, feeder and inland transportation. Our focus is on short-term or medium-term decisions. Thus the long-term decisions such as port capacity choice and investment are fixed and treated as exogenous input variables.

The main contributions of this paper include: (i) an analysis of a novel port competition problem involving both hinterland shipments and transshipment cargoes, by taking the transport chain’s cost perspective including port handling charges, deep sea transport cost, hinterland transport cost, and feeder service cost; (ii) the development of a static cost model for two competitive ports with specific services and analysis of their relative cost in the transport chain, supported by a case study; (iii) the presentation of a non-cooperative game model for two competitive ports and one ocean carrier with multiple shipping services concerning both ports’ pricing decisions and the ocean carrier’s portcall decision. A closed-form of the optimal solution is derived. New managerial insights are obtained, e.g. when the ocean carrier attaches more weight to the congestion cost on either port, then both ports tend

to increase their port prices; (iv) the relative difference between the total profits of the centralized supply chain and the non-cooperative decentralized supply chain is obtained analytically, which quantifies the benefit of the integrated container transport chain; (v) the non-cooperative game model is extended to uncertain demand situations. It is shown that both ports will increase their port handling charges compared to deterministic demand situations; (vi) the results are illustrated using numerical examples based on a case study of Southampton and Liverpool ports.

The rest of the paper is organized as follows. In Section 2, a static cost model is presented for two competitive ports involving hinterland shipments and transshipments with specific shipping services. The purpose is to investigate their relative competitiveness from the transport chain perspective and demonstrate the importance of developing a game model. In Section 3, a non-cooperative game model is formulated for two competitive ports and one ocean carrier with multiple shipping services. The optimal solution is derived and the model is analyzed to generate managerial insights. In Section 4, the associated centralized supply chain model is formulated and compared to the decentralized supply chain model. In Section 5, the non-cooperative game model is extended to uncertain demand situations. In Section 6, a case study is provided to illustrate the results. In Section 7, we provide a discussion for extending the non-cooperative game model to three-port competition situations. Finally, conclusions are presented in Section 8.

## **2. Static cost model of two competitive ports for specific shipping services**

Consider two container ports competing for the same geographic market of hinterland traffic (catchment area) and the same geographic market of transshipment traffic. For example, Liverpool and Southampton ports both serve the UK hinterland market and the transshipment markets including Scotland, Ireland, and Northern Ireland. With the development of the new, deep-water *Liverpool 2* container terminal, these two container ports may compete for the UK hinterland market and the associated transshipment markets.

Suppose an ocean carrier operates a specific deep sea service and a feeder service to serve the container traffic with the option of choosing one of these two ports in its service routes. This gives rise to two alternatives depending on which port is selected. This section analyses the relative cost of these two alternatives. We make the following assumptions:

**Assumption 1.** Both the main service and the feeder service are weekly services. The port times at the two competitive ports are the same. The round-trip journey times of the corresponding service routes (main service or feeder service) for the two alternatives are the same.

**Assumption 2.** At a specific port, the container loading and unloading handling charges are the same. All the containers are measured in TEUs (20-foot equivalent unit). One FEU (40-foot equivalent unit) is treated as two TEUs.

**Assumption 3.** All vessels deployed for the same service are of a similar type.

**Assumption 4.** The number of containers flowing into a port by sea is the same as the number of containers flowing out of the port by sea.

Assumption 1 implies that the same numbers of deep sea vessels (and feeder vessels) are deployed in the two alternatives. Note that one alternative selects one competitive port, whereas the other alternative selects the other competitive port. Therefore, the total sailing distances in the round-trip journey in both alternatives are different. Thus, the planned sailing speeds of the vessels will be different in each alternative in order to maintain the same sailing time at sea in a single round-trip. Assumption 2 is reasonable in the sense that loading and unloading activities (for both laden and empty containers) are very similar. It greatly simplifies the mathematical expressions and makes the results easy to interpret from the practitioners' perspective. However, it should be noted that loading and unloading charges for hinterland shipments could be different from that for transshipment cargoes, particularly at large transshipment ports. Assumption 3 is common in practice because sister vessels are often deployed on the same service route. Assumption 4 follows the common principle of container flow balancing in the literature (e.g. Song and Dong 2013). Note that liner shipping service is a regular service with consecutive round-trip voyages.

We introduce the following notation.

### Notation

- $j$ : index of two competitive ports under consideration, i.e.  $j=1,2$ ;
- $h$ : the hinterland shipment volume in TEUs via two ports in the dominant direction;
- $g$ : the transshipment volume in TEUs via two ports in the dominant direction;
- $\rho$ : the ratio of the portcalls of feeder services to the portcalls of main services at the selected port;
- $c_j^h$ : the unit transportation cost in US\$ in the hinterland associated with port  $j$ ;
- $w_j$ : the unit handling cost in US\$ at port  $j$ ; this is regarded as the price that port  $j$  charges ocean carriers.
- $c^{fuel}$ : the marine fuel cost in US\$ per tonne,
- $s_0^{ms}$ : the designed speed in knots of the vessels in the main service;
- $s_0^{fs}$ : the designed speed in knots of the vessel in the feeder service;
- $s_j^{ms}$ : the sailing speed in knots of the vessels in the main service if calling at port  $j$ ;
- $s_j^{fs}$ : the sailing speed in knots of the vessels in the feeder service if calling at port  $j$ ;
- $d_j^{ms}$ : the sailing distance in nautical miles of the round-trip journey of the main service if calling at port  $j$ ;
- $d_j^{fs}$ : the sailing distance in nautical miles of the round-trip journey of the feeder service if calling at port  $j$ ;
- $FCPD^{ms}$ : the fuel consumption in tonnes per day when a vessel in the main service sails at its designed speed;
- $FCPD^{fs}$ : the fuel consumption in tonnes per day when a vessel in the feeder service sails at its designed speed;
- $G^{ms}(s_j^{ms})$ : the fuel consumption in tonnes per day when a vessel in the main service sails at the speed  $s_j^{ms}$ ;
- $G^{fs}(s_j^{fs})$ : the fuel consumption in tonnes per day when a vessel in the feeder service sails at the speed  $s_j^{fs}$ .

Depending on the transshipment volume and the feeder vessel size, the number of feeder service portcalls is often not equal to the number of main service portcalls. The parameter  $\rho$  is introduced to represent their ratio.

From the transport chain perspective, the total cost includes the following main components: the deployed ship costs (for a time-chartered ship, it refers to the daily charter hire); the fuel

consumption costs by the deployed ships; the port handling charges that incur at ports/terminals; the inland transportation costs (by trucks or trains); the feeder service costs (ships and fuel consumption costs). Note that we are focusing on the cost competitiveness analysis of two alternatives (corresponding to which of two competitive ports is selected in the shipping supply chain). To simplify the narrative, we can exclude the common elements of the costs of each alternative (e.g. the ship costs are the same because the same number of vessels are deployed in both the main service and the feeder service over the same period of time; the port costs, except at the competitive ports under consideration, are the same). Therefore, the total relevant cost in the transport chain associated with one main service port-of-call at port  $j$  is given by,

$$TRC_j = c^{fuel} \cdot d_j^{ms} \cdot G^{ms}(s_j^{ms}) / (24 \cdot s_j^{ms}) + 2(h + g)w_j + 2h \cdot c_j^h + \rho \cdot c^{fuel} \cdot d_j^{fs} \cdot G^{fs}(s_j^{fs}) / (24 \cdot s_j^{fs}) + 2gw_j; \quad (1)$$

Where the first term on the right-hand side of (1) represents the total fuel cost for a vessel sailing the journey distance in the main service (per call at port  $j$ ); the second term is the port handling costs for unloading/loading the hinterland shipments and the transshipments at port  $j$  associated with the main service; the third term is the hinterland transportation costs (forward and backward); the fourth term represents the total fuel cost of the feeder vessel sailing the journey distance in the feeder service (per call at port  $j$ ); the fifth term is the loading/unloading costs at port  $j$  associated with the feeder service.

It should be noted that we did not consider the economy of scale effect with respect to hinterland transport and port handling costs. However, the economy of scale effect for the deep sea main service and feeder service is considered because the fuel consumptions are calculated on a vessel basis rather than on a container basis. The reason for the above treatment is that the economy of scale effect is more prominent in seaborne transport than at port and hinterland.

From Assumption 1, we have the following relationship between the vessel sailing speeds for the two alternatives,

$$s_2^{ms} = d_2^{ms} \cdot s_1^{ms} / d_1^{ms} \text{ and } s_2^{fs} = d_2^{fs} \cdot s_1^{fs} / d_1^{fs} \quad (2)$$

It is commonly accepted that the vessel's fuel consumption has a cubic relationship with its sailing speed. Following the literature, e.g. Ronen (2011), Song and Dong (2013), we assume:  $G^{ms}(s_j^{ms}) = FCPD^{ms} \cdot (s_j^{ms} / s_0^{ms})^3$  and  $G^{fs}(s_j^{fs}) = FCPD^{fs} \cdot (s_j^{fs} / s_0^{fs})^3$ . Together with (1) and (2), we have

$$\begin{aligned} TRC_1 - TRC_2 &= \frac{c^{fuel} \cdot FCPD^{ms} \cdot (s_1^{ms})^2}{24 \cdot (s_0^{ms})^3} \cdot \frac{(d_1^{ms})^3 - (d_2^{ms})^3}{(d_1^{ms})^2} \\ &+ \rho \cdot \frac{c^{fuel} \cdot FCPD^{fs} \cdot (s_1^{fs})^2}{24 \cdot (s_0^{fs})^3} \cdot \frac{(d_1^{fs})^3 - (d_2^{fs})^3}{(d_1^{fs})^2} \\ &+ 2(h + 2g)(w_1 - w_2) + 2h(c_1^h - c_2^h) \end{aligned} \quad (3)$$

**Lemma 1.** Suppose that port 1 has a closer proximity to maritime routes (i.e.  $d_1^{ms} < d_2^{ms}$ ), whereas port 2 has a closer proximity to hinterland markets and to transshipment markets (i.e.  $c_2^h < c_1^h$  and  $d_2^{fs} < d_1^{fs}$ ). We have,

- (i)  $TRC_1 - TRC_2$  is quadratically decreasing in  $s_1^{ms}$ ;
- (ii)  $TRC_1 - TRC_2$  is quadratically increasing in  $s_1^{fs}$ ;

- (iii)  $TRC_1 - TRC_2$  is linearly increasing in  $w_1 - w_2$ ;
- (iv)  $TRC_1 - TRC_2$  is linearly increasing in  $c_1^h - c_2^h$ ;
- (v)  $TRC_1 - TRC_2$  is linearly increasing in  $h$ .

The results in Lemma 1 can be obtained from (3) in a straightforward manner. Port 1 becomes more competitive when the planned sailing speed in the main service route increases, but becomes less competitive when the planned sailing speed in the feed service route increases. Port 2 will be more competitive if it charges less terminal handling fees, or if it has a lower hinterland transport cost than port 1. As the hinterland shipment volume  $h$  increases, port 2 becomes more competitive. The results in Lemma 1 are qualitative. We present a case study to analyse the relative cost competitiveness and sensitivity quantitatively.

### Case study

Consider two competitive ports: Southampton (port 1) and Liverpool (port 2), in which Southampton has a closer proximity to maritime routes, whereas Liverpool may have a closer proximity to hinterland markets (as it is situated centrally in the UK) and to transshipment markets (closer to Ireland and Scotland). Notteboom et al. (2014) provided historical data in terms of transshipment market share at Southampton and Liverpool, which shows Southampton has transshipment percentage at 6.0% in 2004 and 5.5% in 2012; whereas Liverpool has transshipment percentage at 6.2% in 2004 and 8.0% in 2012. In the base scenario, we assume that the hinterland shipment volume is 1000 TEUs and the transshipment volume is 100 TEUs (about 10% transshipment market share). It should be noted that there are more main (deep sea) services calling at Southampton than feeder services due to the low transshipment volume and the deployment of feeder vessels with capacity 500 ~ 1000 TEUs. Approximately, five main service portcalls correspond to one feeder service portcall. Thus, we take the ratio  $\rho=1/5$  in our calculation.



Figure 1. Graphic illustrations of the network configurations with deep sea service AX1 (bottom) and feeder service SLX II (top) (based on NYK Lines and X-Press Lines)

In Alternative 1, we select AX1 service (operated by Grand Alliance: [www2.nykline.com/liner/service\\_network/ax1.html](http://www2.nykline.com/liner/service_network/ax1.html)) as the main deep sea service, which has the port rotation: Le Havre -> Rotterdam -> Hamburg -> Southampton -> New York -> Norfolk -> Charleston -> Le Havre. It has a journey distance  $d^{ms}_1 = 8556$  nautical miles. Five vessels are deployed to provide a weekly service in this main service route with each vessel having a capacity of 8750 TEUs. The feeder service is the SLX II service (operated by X-Press Container Lines: [www.dpworldsouthampton.com](http://www.dpworldsouthampton.com)) with the port rotation: Southampton -> Dublin -> Belfast -> Greenock -> Southampton. It has a journey distance  $d^{fs}_1 = 1141$  nautical miles. One vessel with the capacity 900 TEUs is deployed to provide the weekly feeder service. Figure 1 provides a graphic illustration of the network configurations with deep sea service AX1 and feeder service SLX II.

In Alternative 2, the main service and feeder service have the same port rotation as Alternative 1 except that Southampton is replaced with Liverpool. The journey distance of the main service  $d^{ms}_2 = 8949$  nautical miles, and the journey distance of the feeder service  $d^{fs}_2 = 517$  nautical miles.

The system parameters are set up as follows (based on Carou 2011, Song and Dong 2013, and Pocuca 2006): the fuel price  $c^{fuel} = 400$  US\$/tonne; the vessel's designed speed  $s^{ms}_0 = 24.6$  knots in the main service and  $s^{fs}_0 = 19$  knots in the feeder service; the daily fuel consumption



of the vessel sailing at the designed speed  $FCPD^{ms} = 272$  tonnes in the main service, and  $FCPD^{fs} = 50$  tonnes in the feeder service. We take the vessel planned sailing speed  $s_1^{ms} = s_1^{fs} = 18$  knots, which is a common sailing speed due to the adoption of slow steaming. In the following, we conduct two groups of experiments to examine the impact of system parameters (or decisions) on the cost competitiveness of the two alternatives.

In the first group, we assume that two ports have the same terminal handling charge, i.e.  $w_1 = w_2$ . We vary the difference of their hinterland transportation costs, e.g. let  $c_1^h - c_2^h$  take values from \$0, \$20, \$40, \$60, \$80 respectively. This represents the fact that Liverpool is relatively closer to the hinterland markets than Southampton. The hinterland shipment volume  $h$  ranges from 1000 TEUs to 1200, 1400, and 1600 TEUs. Figure 2 shows the results of the experiments, in which each cluster of the bars corresponds to different levels of the hinterland shipment volume. It can be observed that when the difference between the hinterland transportation costs is less than \$20, Alternative 1 is more cost efficient. When  $c_1^h - c_2^h$  is greater than \$40 and the hinterland shipment volume is greater than 1400 TEUs, Alternative 2 is more cost efficient. This implies that larger hinterland markets would be in favour of shipping services calling at Liverpool.

In the second group, we assume Liverpool port's terminal handling charge is \$20 less than Southampton's. The relevant cost differences between two alternatives with varying  $c_1^h - c_2^h$  and  $h$  are shown in Figure 3. It can be observed that Alternative 2 now becomes more competitive than Alternative 1 in most scenarios. Note that in the current shipping market, the shipment volume is relatively low. Liverpool port is further away from the maritime routes than Southampton, it is reasonable for Liverpool port (and the shipping lines) to lower the terminal handling charge to be competitive. This reflects the current practice, e.g. CMA CGM's terminal handling charge at Liverpool is £15 (about 22 US\$) less than that at Southampton.

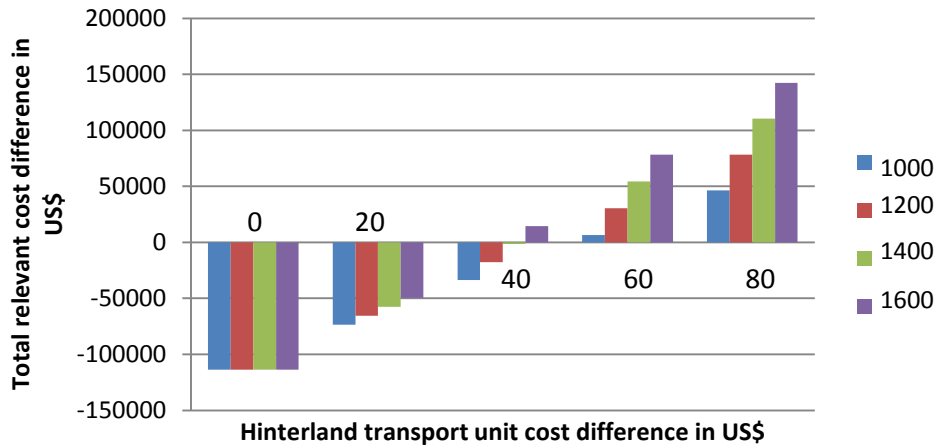


Figure 2. Total relevant cost difference ( $TRC_1 - TRC_2$ ) with  $w_1 = w_2$

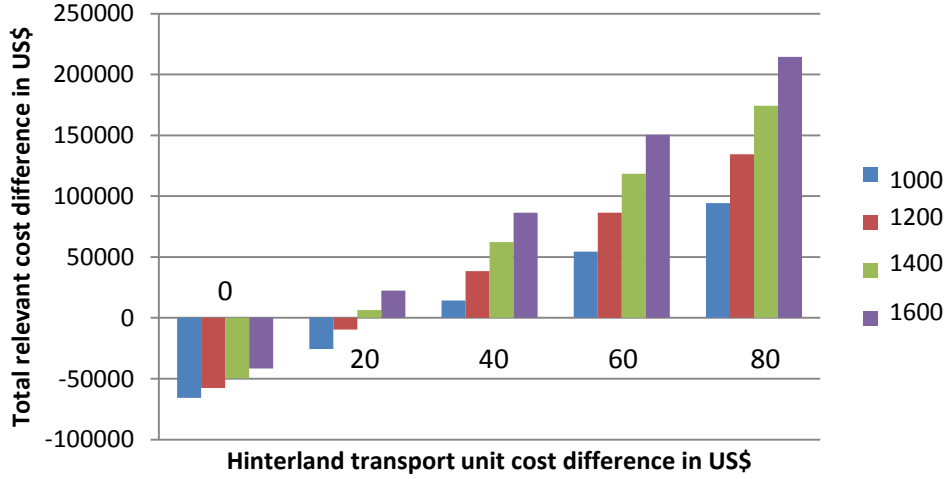


Figure 3. Total relevant cost difference ( $TRC_1 - TRC_2$ ) with  $w_1 - w_2 = \$20$

The above case study illustrates that either of the two neighbouring ports can be more cost efficient under certain conditions, or by appropriately adjusting their terminal handling charge. Together with other experiments, we observed that higher planned vessel sailing speeds and higher fuel cost would favour Southampton's selection, whereas higher hinterland shipment and higher transshipment volumes would favour Liverpool. In particular, the port handling charges at the two ports have a significant impact on their relative competitiveness. Therefore, it is more likely that each of the competitive ports can attract a fraction of total shipping services.

The cost model in (3) provides a simple way to analyze the relative competitiveness of two competitive ports for specific shipping supply chains. However, the model is static and does not take into account the issues such as: ports' interactive decisions on pricing (as game scenarios), port capacity and congestion, and the carrier's multiple service decisions (e.g. the ports-of-call split between two competitive ports) influenced by the port pricing and port congestion. In particular, when one port changes its port charge, the other port may respond. This may affect the ocean carrier's decision of portcalls since the port handling charge is one key component of the total transport chain cost. Therefore, it is important to model the port competition within a game framework in the transport chain context. The remainder of this paper will extend the static cost model into a game framework and allow the portcalls to be split between two competitive ports in response to the ports' pricing decisions.

### 3. Game cost model for two ports and one carrier with multiple services

Consider a system consisting of a single shipping line (or an alliance) and two competitive container ports ( $j=1, 2$ ), in which both ports serve the same (or partially overlapped) hinterland market and the same transshipment markets. The focus on a single shipping line can be justified as follows. Firstly, major shipping lines have formed into different alliances, e.g. M2, Ocean3, G6 and CKYHE. The alliance may be treated as an aggregated shipping line from the operational perspective. Secondly, dedicated container terminals normally service a single shipping line. Thirdly, a single shipping line case is easy to analyze technically and can provide intuitive managerial insights for practitioners.

The total amount of hinterland containers via the two ports is assumed fixed, but their split between the two ports is proportional to their split of vessel portcalls. Similarly, it is assumed

that the total amount of transshipment containers via the two ports is fixed, but the transshipment volume via each port depends on the number of vessel portcalls at the corresponding port. This assumption is based on Bae et al. (2013). It should be noted that, in practice, the split of shipments may be affected by other factors such as terminal handling charge and service level. In our study, both the main deep sea service and feeder service are operated by the aggregated shipping line (most global shipping lines operate both deep sea services and feeder services, or have a long-term contract with feeder operators). We introduce the notation for the game model as follows:

**Additional notation:**

- $p^h$ : the hinterland shipment unit price (revenue) in US\$; we assume that the return shipment price has been factored into  $p^h$ ;
- $p^t$ : the transshipment unit price (revenue) in US\$; we assume that the return transshipment price has been factored into  $p^t$ ;
- $q_j$ : the fraction of vessel portcalls at port  $j$ ; Here  $q_j$  is the shipping line's decision variable such that  $0 \leq q_1 \leq 1$  and  $q_2 = 1 - q_1$ , which may depend on port handling capacity, port congestion, port prices, hinterland transportation costs, transshipment costs, and other relevant cost parameters;
- $c_j^s$ : the deep sea vessel fuel cost at sea per port-of-call at port  $j$ ; this can be defined as the first term in Eq. (1);
- $c_j^f$ : the feeder vessel fuel cost at sea per port-of-call at port  $j$ ; this can be defined as the fourth term in Eq. (1);
- $F_j$ : the number of containers in TEUs that are handled at port  $j$  (loading and unloading are counted separately);
- $K_j$ : the effective handling capacity at port  $j$ ;
- $a_j$ : a positive coefficient, representing the congestion cost in US\$ when the utilization of port  $j$  reaches its effective capacity;
- $R_j$ : the effective hinterland transport capacity at port  $j$ ;
- $b_j$ : a positive coefficient, representing the congestion cost in US\$ when the utilization of hinterland transport of port  $j$  reaches its effective capacity;
- $c_j$ : the unit operating cost at port  $j$ ;
- $m_j$ : the unit handling capacity investment at port  $j$ ;

It is assumed that the competitive shipments via each port are proportional to the fraction of portcalls at the corresponding port (Bae et al. 2013). The shipment ( $g$  and  $h$ ) in our study refers to the laden containers in the dominant direction. Under Assumption 4, vessels and ports have to handle the container flows in both directions. More specifically, an importing transshipment container will first be lifted off from the mother vessel to the port; then lifted onto a feeder vessel from the port. After the feeder vessel reaches the destination port, the container will be discharged and unpacked and becomes empty. The empty container may be reloaded with new goods for export or returned as an empty container; the returned container will then be lifted off from the feeder vessel to the port; then lifted onto a mother vessel from the port. Therefore, one transshipment container implies four lifts in total. It follows,

$$F_j = 2(h + 2g) \cdot q_j, \text{ for } j=1,2;$$

The port congestion cost is assumed to be a function as follows,

$$G_j = a_j \cdot (F_j / K_j)^n = a_j \cdot [2(h + 2g) \cdot q_j / K_j]^n, \text{ for } j=1,2 \quad (4)$$

Normally, we would have  $F_j \leq K_j$ . Hence,  $F_j / K_j$  can be regarded as the port  $j$ 's utilization. The coefficient  $n \geq 1$  indicates the relationship between the port congestion cost and the port utilization. If  $n=1$ , then the congestion cost is linear to its utilization (e.g. De Borger and Van Dender 2006). If  $n=2$ , then the relationship is quadratic (e.g. Bae et al. 2013). Similarly, the hinterland transport congestion cost can be defined as

$$H_j = b_j \cdot [2h \cdot q_j / R_j]^n, \text{ for } j=1,2 \quad (5)$$

The above function can be regarded as a simplified model of De Borger et al. (2008) by ignoring the non-container local traffic that share the hinterland transport road/ railway. It is noted that including non-container local traffic would complicate the narrative and discussion, but would not affect the main results.

The shipping line's profit function is given by,

$$\pi^l = \sum_j [(p^h - 2c_j^h - 2w_j) \cdot h \cdot q_j + (p^t - 4w_j) \cdot g \cdot q_j - c_j^s \cdot q_j - c_j^t \cdot q_j - G_j - H_j] \quad (6)$$

s.t.

$$0 \leq q_1, q_2 \leq 1; \text{ and } q_2 = 1 - q_1$$

The ports' profit functions (for  $j=1,2$ ) are given by,

$$\pi_j = (w_j - c_j) \cdot F_j - m_j \cdot K_j = (w_j - c_j) \cdot 2(h + 2g) \cdot q_j - m_j \cdot K_j \quad (7)$$

In practice, ports' pricing decisions are bounded, denoted by  $L_j \leq w_j \leq U_j$ .

### 3.1 The non-cooperative game model

In a non-cooperative game, each player makes decisions independently. It can be formulated as a two stage problem. At the first stage, each port makes port handling pricing decisions to maximize its profit. At the second stage, the shipping line makes portcall decisions to maximize its profit by observing the ports' congestion, prices, transshipment level, hinterland shipment level, deep sea vessel operating cost, hinterland transportation cost, and feeder vessel operating cost.

To solve the problem, the backwards induction approach is used (e.g. Bae et al. 2013). The approach can be summarized as follows: (i) For the second stage, the sub-game Nash equilibrium can be obtained. The portcall decision variables can be represented as a function of port capacities, prices, transshipment and hinterland shipment levels; (ii) For the first stage, by utilizing the portcall decisions obtained at the second stage, the Nash equilibrium port prices can then be derived; (iii) Finally, the optimal port prices would yield the shipping line's portcall decisions.

From Eqs. (4)~(6), we have,

$$\begin{aligned} \pi^l = & [(p^h - 2c_1^h - 2w_1) \cdot h \cdot q_1 + (p^t - 4w_1) \cdot g \cdot q_1 - c_1^s \cdot q_1 - c_1^t \cdot q_1] \\ & + [(p^h - 2c_2^h - 2w_2) \cdot h \cdot (1 - q_1) + (p^t - 4w_2) \cdot g \cdot (1 - q_1) - c_2^s \cdot (1 - q_1) - c_2^t \cdot (1 - q_1)] \\ & - \frac{a_1(2(h + 2g)q_1)^n}{K_1^n} - \frac{a_2(2(h + 2g)(1 - q_1))^n}{K_2^n} - \frac{b_1(2hq_1)^n}{R_1^n} - \frac{b_2(2h(1 - q_1))^n}{R_2^n} \end{aligned} \quad (8)$$

**Lemma 2.** For the given port prices  $w_1$  and  $w_2$ , the optimal profit of the shipping line,  $\pi^l$ , is concave with respect to  $q_1$  in the interval  $[0, 1]$ .

For narrative and discussion expediency, we have placed the proofs of all lemmas and propositions in the Appendix with those regarded as straightforward being simplified or omitted.

Lemma 2 indicates that for the given port prices  $w_1$  and  $w_2$ ,  $\partial\pi/\partial q_1$  is a monotonic decreasing function in  $q_1$  in the interval  $[0, 1]$ . Therefore, there is a unique, optimal solution  $q_1^*$  in the interval  $[0, 1]$ . More specifically,  $q_1^* = 0$  if  $\partial\pi/\partial q_1 < 0$  for any  $q_1 \in [0, 1]$ ;  $q_1^* = 1$  if  $\partial\pi/\partial q_1 > 0$  for any  $q_1 \in [0, 1]$ ; and  $q_1^* \in (0, 1)$ , otherwise.

Note that in Lemma 2, the port prices have not been optimized. Next, we will examine two ports' decisions by taking into account the shipping line's behaviour. To make the problem analytically tractable and explore more managerial insights, we consider the case with  $n = 2$  in the rest of the paper.

### 3.2 The optimal solution

To simplify the narrative, let  $A_1 := 8a_1 \cdot (h + 2g)^2/K_1^2 + 8b_1 \cdot h^2/R_1^2$ ;  $A_2 := 8a_2 \cdot (h + 2g)^2/K_2^2 + 8b_2 \cdot h^2/R_2^2$ ; and  $B := 2(c_2^h - c_1^h) \cdot h + c_2^s - c_1^s + c_2^t - c_1^t$ . The portcall decision variables can be represented as a function of port prices as follows.

**Lemma 3.** For the given port prices  $w_1$  and  $w_2$ , the shipping line's optimal portcall decisions are given by

$$q_1^* = \begin{cases} 0 & D_1 < 0 \\ D_1 & 0 \leq D_1 \leq 1; \\ 1 & D_1 > 1 \end{cases}$$

and  $q_2^* = 1 - q_1^*$ ; where  $D_1 := [B + A_2 + 2(w_2 - w_1) \cdot (h + 2g)] / (A_1 + A_2)$ .

By utilizing the portcall decisions in Lemma 3 (at the second stage), the Nash equilibrium port prices can then be derived at the first stage, which then leads to the shipping line's optimal portcall decisions. The following proposition summarizes the main results.

**Proposition 1.** The optimal decisions of the shipping line and two ports are given by (with  $q_2^* = 1 - q_1^*$ ):

- (i) if  $D_1 < 0$ , then  $q_1^* = 0$ ;  $w_1^*$  and  $w_2^*$  are given by  
 $(w_1^*, w_2^*) = \max\{(w_1, w_2) \mid L_1 \leq w_1 \leq U_1, L_2 \leq w_2 \leq U_2 \text{ and } w_2 - w_1 < (-A_2 - B)/(2h + 4g)\}$
- (ii) if  $D_1 > 1$ , then  $q_1^* = 1$ ;  $w_1^*$  and  $w_2^*$  are by  
 $(w_1^*, w_2^*) = \max\{(w_1, w_2) \mid L_1 \leq w_1 \leq U_1, L_2 \leq w_2 \leq U_2 \text{ and } w_2 - w_1 > (A_1 - B)/(2h + 4g)\}$
- (iii) if  $0 \leq D_1 \leq 1$ ,  $L_1 \leq w_1 \leq U_1$ , and  $L_2 \leq w_2 \leq U_2$ , then  $w_1^*$ ,  $w_2^*$ , and  $q_1^*$  are given by

$$w_1^* = \frac{(A_1 + 2A_2) + B}{6(h + 2g)} + \frac{2c_1 + c_2}{3} \quad (9)$$

$$w_2^* = \frac{(2A_1 + A_2) - B}{6(h + 2g)} + \frac{c_1 + 2c_2}{3} \quad (10)$$

$$q_1^* = \frac{A_1 + 2A_2 + B + 2(c_2 - c_1) \cdot (h + 2g)}{3(A_1 + A_2)} \quad (11)$$

Proposition 1 provides the optimal decisions of the two ports and the shipping line in the non-cooperative game system. Proposition 1(i) and (ii) represent the cases that one of the ports

will gain no business in the competitive hinterland and transshipment markets. This is unusual in practice as we explained in the static cost model. Proposition 1(iii) is a more interesting case, in which the optimal decisions do not take the boundary values. It provides the Nash equilibrium solution to the non-cooperative game. The three conditions in Proposition 1(iii) can be replaced by the following explicit inequalities (cf. the proof of Proposition 1):

$$-A_2 \leq \frac{(A_1 - A_2) + B + 2(c_2 - c_1)(h + 2g)}{3(h + 2g)} \leq A_1 \quad (12)$$

$$L_1 \leq \frac{(A_1 + 2A_2) + B}{6(h + 2g)} + \frac{2c_1 + c_2}{3} \leq U_1 \quad (13)$$

$$L_2 \leq \frac{(2A_1 + A_2) - B}{6(h + 2g)} + \frac{c_1 + 2c_2}{3} \leq U_2 \quad (14)$$

**Proposition 2.** Under the conditions (12)~(14), the optimal profit functions for the two ports and the shipping line are given by:

$$\begin{aligned} \pi_1^* &= \frac{[A_1 + 2A_2 + B + 2(c_2 - c_1)(h + 2g)]^2}{9(A_1 + A_2)} - m_1 K_1 \\ \pi_2^* &= \frac{[2A_1 + A_2 - B + 2(c_1 - c_2)(h + 2g)]^2}{9(A_1 + A_2)} - m_2 K_2 \\ \pi^{ls} &= (p^h - 2c_2^h - 2w_2^*)h + (p^t - 4w_2^*)g - c_2^s - c_2^t \\ &\quad + (B + 2(w_2^* - w_1^*)(h + 2g))q_1^* - A_1 q_1^{*2} / 2 - A_2 (1 - q_1^*)^2 / 2 \end{aligned}$$

Where  $w_1^*$  and  $w_2^*$  are given in (9) and (10);  $q_1^*$  is given in (11).

### 3.3 Analysis of the optimal solutions

Now we analyse the impact of important input parameters on the optimal decisions of the two ports and the ocean carrier in the non-cooperative game context.

**Lemma 4.** Under the conditions (12)~(14), the impact of the hinterland shipment and the transshipment volume on decision variables is given by:

- (i)  $\partial w_1^* / \partial h = [A_1 + 2A_2 + 32b_1 \cdot hg / R_1^2 + 64b_2 \cdot hg / R_2^2 + 2(c_2^h - c_1^h)(h + 2g) - B] / [6(h + 2g)^2]$
- (ii)  $\partial w_1^* / \partial g = [A_1 + 2A_2 - 16b_1 \cdot h^2 / R_1^2 - 32b_2 \cdot h^2 / R_2^2 - B] / [3(h + 2g)^2]$
- (iii)  $\partial w_2^* / \partial h = [2A_1 + A_2 + 64b_1 \cdot hg / R_1^2 + 32b_2 \cdot hg / R_2^2 - 2(c_2^h - c_1^h)(h + 2g) + B] / [6(h + 2g)^2]$
- (iv)  $\partial w_2^* / \partial g = [2A_1 + A_2 - 32b_1 \cdot h^2 / R_1^2 - 16b_2 \cdot h^2 / R_2^2 + B] / [3(h + 2g)^2]$

From Lemma 4, it can be seen that: (a) note that  $2(c_2^h - c_1^h)(h + 2g) - B = 4(c_2^h - c_1^h)g - c_2^s + c_1^s - c_2^t + c_1^t$ . If  $4(c_2^h - c_1^h)g - c_2^s + c_1^s - c_2^t + c_1^t \geq 0$ , then we always have:  $\partial w_1^* / \partial h > 0$ . This implies that  $w_1^*$  is strictly increasing in  $h$ . If  $4(c_2^h - c_1^h)g - c_2^s + c_1^s - c_2^t + c_1^t < 0$ , then  $\partial w_1^* / \partial h$  is strictly increasing in  $h$  and converges to a finite positive number. Thus, there exists a threshold value  $h_1^* \geq 0$ , when  $h \geq h_1^*$ ,  $\partial w_1^* / \partial h$  is always greater than 0. The implication is that port 1's price will increase in  $h$  when  $h$  reaches a certain level. (b) If  $A_1 + 2A_2 - 16b_1 \cdot h^2 / R_1^2 - 32b_2 \cdot h^2 / R_2^2 - B \geq 0$ , then  $\partial w_1^* / \partial g$  is greater than 0, i.e.  $w_1^*$  is increasing in  $g$ . If  $A_1 + 2A_2 - 16b_1 \cdot h^2 / R_1^2 - 32b_2 \cdot h^2 / R_2^2 - B < 0$ , then  $\partial w_1^* / \partial g$  is increasing in  $g$  and converges to a finite positive number; Thus, there exists a threshold value  $g_1^* \geq 0$ , when  $g \geq g_1^*$ ,  $\partial w_1^* / \partial g$  is always greater than 0. Similar interpretations to port 2's pricing decisions with respect to  $h$  and  $g$  can be obtained from assertions (iii) and (iv). Finally, the expressions of the partial derivative of

the ocean carrier's portcall decision with respect to the hinterland volume and the transshipment volume are complicated and have been omitted.

The above results provide the relationships between the players' (two ports and the ocean carrier) decisions and the hinterland volume and the transshipment volume. In summary, it is not guaranteed that ports' prices will increase in the hinterland shipments or the transshipment volume. However, as  $g$  and  $h$  reach certain threshold levels, the two ports' handling prices are indeed increasing in  $g$  and  $h$ . This is intuitive since sufficiently high hinterland shipments or transshipment volumes would encourage both ports to raise their handling charges. However, the interesting point here is that we provide a simple formula to analytically determine the threshold levels that are able to characterize the ports' pricing behaviours.

Consider the case of Southampton (port 1) and Liverpool (port 2). Since Southampton has a closer proximity to maritime routes, whereas Liverpool may have a closer proximity to transshipment markets (closer to Ireland and Scotland), taking into account the vessel sizes in the deep sea and feeder services, we have:  $c_2^s - c_1^s + c_2^t - c_1^t > 0$  (based on the case study in Section 2). Assume that  $c_2^h = c_1^h$ ,  $c_2 = c_1$ ,  $a_2 = a_1$ ,  $K_2 = K_1$ ,  $b_2 = b_1$ ,  $R_2 = R_1$ , i.e. the two ports have similar unit hinterland transport cost, unit port handling cost, congestion coefficients, port capacity and hinterland transport capacity. From Lemma 4(i) and (iii), it follows:  $\partial w_1^*/\partial h < \partial w_2^*/\partial h$ . Because in general we have  $\partial w_1^*/\partial h > 0$ , this implies that Liverpool's port price is increasing more quickly than Southampton as the hinterland shipment volume increases. A similar pattern can be observed with respect to the transshipment volume. The implication is that Liverpool may become more competitive when the hinterland or transshipment market is larger. Thus, a strategy to expand market size is more desirable and important for the Liverpool port to compete with Southampton. Our model provides knowledge of the relationship between the shipment market size and the relative competitiveness of the two ports.

**Lemma 5.** Under the conditions (12)~(14), the impact of key cost parameters on decision variables is given by:

- (i)  $\partial w_1^*/\partial c_1 = \partial w_2^*/\partial c_2 = 2/3$ ;
- (ii)  $\partial w_2^*/\partial c_1 = \partial w_1^*/\partial c_2 = 1/3$ ;
- (iii)  $\partial q_1^*/\partial c_1 = -2(h+2g)/[3(A_1 + A_2)]$ , and  $\partial q_1^*/\partial c_2 = 2(h+2g)/[3(A_1 + A_2)]$ ;
- (iv)  $\partial w_1^*/\partial c_1^s = \partial w_2^*/\partial c_2^s = -1/[6(h+2g)]$ ;  $\partial w_1^*/\partial c_2^s = \partial w_2^*/\partial c_1^s = 1/[6(h+2g)]$ ;
- (v)  $\partial q_1^*/\partial c_1^s = -1/[3(A_1 + A_2)]$ , and  $\partial q_1^*/\partial c_2^s = 1/[3(A_1 + A_2)]$ ;
- (vi)  $\partial w_1^*/\partial c_1^h = \partial w_2^*/\partial c_2^h = -h/[3(h+2g)]$ ;  $\partial w_1^*/\partial c_2^h = \partial w_2^*/\partial c_1^h = h/[3(h+2g)]$ ;
- (vii)  $\partial q_1^*/\partial c_1^h = -2h/[3(A_1 + A_2)]$ , and  $\partial q_1^*/\partial c_2^h = 2h/[3(A_1 + A_2)]$ ;
- (viii)  $\partial w_1^*/\partial c_1^t = \partial w_2^*/\partial c_2^t = -g/[6(h+2g)]$ ;  $\partial w_1^*/\partial c_2^t = \partial w_2^*/\partial c_1^t = g/[6(h+2g)]$ ;
- (ix)  $\partial q_1^*/\partial c_1^t = -g/[3(A_1 + A_2)]$ , and  $\partial q_1^*/\partial c_2^t = g/[3(A_1 + A_2)]$ ;

Lemma 5(i)~(iii) reveals that: the two ports' optimal prices are both increasing with constant rates as either port's unit operating cost (i.e.  $c_j$ ) increases. However, the port's price is more sensitive to its own operating cost than to the other port's operating cost. In addition, the shipping line's optimal portcall fraction is decreasing in the corresponding port's operating cost, which is reasonable since the port's handling charge is increasing. It should be noted that the ports' prices and unit operating cost are influenced by port development doctrines (Lee and Flynn 2011), which may vary between regions.

Lemma 5(iv)~(v) reveals that: port 1's price will be decreasing as the fuel cost at deep sea service associated with port 1 ( $c_1^s$ ) increases, and will be increasing at the same rate if the fuel cost at deep sea service associated with port 2 ( $c_2^s$ ) increases. The shipping line's portcall fraction to port 1 is decreasing when  $c_1^s$  increases, or  $c_2^s$  decreases. Physically, if port 1 has a closer proximity to maritime routes (with lower  $c_1^s$ ), port 2 tends to reduce its handling charge to attract the shipping line's portcall.

Lemma 5(vi)-(vii) reveals that if port 1 is further away from the hinterland market (with higher  $c_1^h$ ), then port 1 tends to reduce its handling charge, whereas port 2 is able to increase its handling charge. The shipping line would increase the portcall fraction to the port with closer proximity to the hinterland market. Similar phenomena can be observed in terms of the impact of the fuel cost of the feeder vessels (i.e.  $c_1^f$ ) on the ports' pricing decisions and the shipping line's portcall decisions.

Consider the case of Southampton (port 1) and Liverpool (port 2). Note that Liverpool may have a closer proximity to hinterland markets (as it is situated centrally in the UK). This can be represented by  $c_2^h < c_1^h$ . From Lemma 5(vi)-(vii), as Southampton's hinterland transport cost  $c_1^h$  increases, the optimal port price at Southampton  $w_1^*$  is decreasing, the optimal port price at Liverpool  $w_2^*$  is increasing, and the optimal portcall fraction at Southampton is decreasing. The implication is that Liverpool has a disadvantage of longer distance from maritime routes, which makes its port price less competitive than Southampton (e.g. CMA CGM's terminal handling charge at Liverpool is £15 less than that at Southampton). However, its close proximity to hinterland markets can improve its competitiveness, reduce the price gap, and even outperform Southampton in terms of the portcall fraction as shown in Figure 7.

**Lemma 6.** Under the conditions (12)~(14), the impact of port congestion cost and handling capacity on decision variables is given by:

- (i)  $\partial w_1^* / \partial a_1 = 4(h + 2g) / (3K_1^2)$ ;  $\partial w_2^* / \partial a_1 = 8(h + 2g) / (3K_1^2)$ ;
- (ii)  $\partial q_1^* / \partial a_1 = -8[A_2 + B + 2(c_2 - c_1)(h + 2g)](h + 2g)^2 / [3K_1^2(A_1 + A_2)^2]$ ;
- (iii)  $\partial w_1^* / \partial a_2 = 8(h + 2g) / (3K_2^2)$ ;  $\partial w_2^* / \partial a_2 = 4(h + 2g) / (3K_2^2)$ ;
- (iv)  $\partial q_1^* / \partial a_2 = 8[A_1 - B - 2(c_2 - c_1)(h + 2g)](h + 2g)^2 / [3K_2^2(A_1 + A_2)^2]$ ;
- (v)  $\partial w_1^* / \partial K_1 = -8(h + 2g)a_1 / (3K_1^3)$ ;  $\partial w_2^* / \partial K_1 = -16(h + 2g)a_1 / (3K_1^3)$ ;
- (vi)  $\partial q_1^* / \partial K_1 = 16[A_2 + B + 2(c_2 - c_1)(h + 2g)]a_1(h + 2g)^2 / [3K_1^3(A_1 + A_2)^2]$ ;
- (vii)  $\partial w_1^* / \partial K_2 = -16(h + 2g)a_2 / (3K_2^3)$ ;  $\partial w_2^* / \partial K_2 = -8(h + 2g)a_2 / (3K_2^3)$ ;
- (viii)  $\partial q_1^* / \partial K_2 = -16[A_1 - B - 2(c_2 - c_1)(h + 2g)]a_2(h + 2g)^2 / [3K_2^3(A_1 + A_2)^2]$ ;
- (ix)  $\partial w_1^* / \partial R_1 = -8b_1 \cdot h^2 / [3R_1^3(h + 2g)]$ ;  $\partial w_2^* / \partial R_1 = -16b_1 \cdot h^2 / [3R_1^3(h + 2g)]$ ;
- (x)  $\partial q_1^* / \partial R_1 = 16[A_2 + B + 2(c_2 - c_1)(h + 2g)]b_1 \cdot h^2 / [3R_1^3(A_1 + A_2)^2]$ ;

Note that the coefficient  $a_j$  can be regarded as the weight that the shipping line places on the congestion situation at port  $j$ . From Lemma 6, it can be seen that: (a) the two ports' prices are increasing as either  $a_1$  or  $a_2$  increases. In addition, port 1's price is more sensitive to the congestion cost coefficient at port 2 than that at port 1. As  $a_1$  increases, the shipping line's portcall fraction at port 1,  $q_1^*$ , will be decreasing if  $A_2 + B + 2(c_2 - c_1)(h + 2g) > 0$ ; (b) both ports' prices are decreasing as either port's effective handling capacity (i.e.  $K_j$ ) is increasing. Port 1's price is more sensitive to the handling capacity at port 2 than that at port 1. In addition, as  $K_1$  increases, the shipping line's portcall decision  $q_1^*$  will be increasing if  $A_2 + B + 2(c_2 - c_1)(h + 2g) > 0$ ; (c) both ports' prices are decreasing as either port's hinterland transport capacity (i.e.  $R_j$ ) is increasing. Port 1's price is more sensitive to the hinterland



transport capacity at port 2 than that at port 1. In addition, as  $R_1$  increases, the shipping line's portcall decision  $q_1^*$  will be increasing if  $A_2 + B + 2(c_2 - c_1)(h+2g) > 0$ .

Two interesting and revealing phenomena can be observed. Intuitively, when the shipping line attaches more weight to the congestion cost at port 1 (by increasing  $a_1$ ), port 1 may decrease its price to attract the shipping line's portcall to cancel out the impact of the higher congestion cost. However, Lemma 6(i) indicates that port 1 will actually increase its price. This behaviour may be explained by the fact that when the shipping line attaches more weight to the congestion cost, the shipping line becomes more willing to accommodate relatively higher port prices for a less congested port. More specifically, suppose  $a_1$  is increased. This breaks the current Nash equilibrium and the shipping line may shift some portcall from port 1 to port 2. Such shift enables port 2 to increase its port price, which then results in the increase of price at port 1 to reach a new Nash equilibrium. This also explains why port 2's price is more sensitive to  $a_1$  than port 1's price. The above discussion is based on an implicit assumption that the shipping line bears the congestion costs (e.g. in the situations of door-to-door service, the shipping line is responsible for the entire journey of the container movements). However, in practice shipping lines may impose a congestion surcharge to shippers to compensate for the incurred port congestion cost for those ports that are congested regularly, in which case the model should be extended to include shippers as one of the decision makers.

The second phenomenon is that when a port's handling capacity increases, the ports' prices are decreasing. Intuitively, as ports' handling capacity increases, they become more attractive due to less congestion and therefore ports could increase their port prices to generate more revenue. However, this counter-intuitive behaviour may be explained as follows: suppose port 1 increases the handling capacity; this will break the current Nash equilibrium and result in a portcall shift from port 2 to port 1. Such a shift may drive port 2 to decrease the port price, which leads to the reduction of port price at port 1 in order to maintain competitiveness. Eventually, a new Nash equilibrium will be achieved. This can also explain why port 2's price is more sensitive to  $K_1$  than port 1's price. We can observe that when port 1 increases its handling capacity, it can generally attract more portcalls and cargoes. This in turn increases the congestion of the hinterland transport at port 1, which may affect the hinterland shipment volume. Lemma 6(vi) and (viii) represent such interactions. In addition, as port 1 increases its handling capacity, port 1's congestion will decrease if and only if  $K_1 \cdot \partial q_1^* / \partial K_1 < q_1^*$ .

In terms of the portcall decision with respect to the congestion cost coefficient, note that  $A_2 > 0$  and  $B + 2(c_2 - c_1)(h+2g) = 2(c_2^h - c_1^h) \cdot h + (c_2^s - c_1^s + c_2^t - c_1^t) + 2(c_2 - c_1)(h+2g)$ ; if two ports have the similar capacity investment cost per unit (i.e.  $c_2 = c_1$ ), similar hinterland transportation cost (i.e.  $c_2^h = c_1^h$ ), and port 1 is closer to the maritime route overall (i.e.  $c_2^s - c_1^s + c_2^t - c_1^t > 0$ ), then Lemma 6(ii) indicates that the portcall fraction at port 1 will be decreasing as port 1's congestion cost coefficient (i.e.  $a_1$ ) increases. This implies that although port 1 is more competitive than port 2, the shipping line will still shift some portcalls to port 2 if the congestion concern increases at port 1.

#### 4. The centralized management model for two ports and one carrier

In the previous section, we considered a decentralized supply chain in which each player (the shipping line or either of two ports) is making decisions to maximize its own profit. In this section we consider a centralized supply chain in which the overall supply chain profit can be maximized.

Investigating the centralized scenario is helpful even for a single shipping line case. This may be explained by the following. First, a centralized management scenario implies vertical integration between ports and the shipping line, which is an important strategy to cut operational costs and compete with other carriers or shipping supply chains. In practice, many global shipping lines are also the terminal operators and/or have leased dedicated container terminals to achieve vertical integration to some extent. Second, the coordination between two ports (modelled in the centralized scenario) is of practical interest. For example, Ningbo port and Zhoushan port have been centrally managed by the local government. Cooperation between Hong Kong and Yantian is achieved through the HPH Group's common ownership (Song 2002).

In the centralized management model, ports' prices are internalized and ports' demands are mainly affected by port capacity, hinterland shipment volume, transshipment volume, and the relevant costs associated with the ports. Under the condition  $0 \leq D_1 \leq 1$ , which ensures that  $q_1^* \in [0, 1]$ , the supply chain profit for a given set of decisions  $(w_1, w_2, q_1)$  is defined by,

$$\begin{aligned} \pi(w_1, w_2, q_1) &= \pi_1(w_1, w_2, q_1) + \pi_2(w_1, w_2, q_1) + \pi^l(w_1, w_2, q_1) \\ &= (p^h - 2c^h_2)h + p^t \cdot g - c^s_2 - c^t_2 - 2c_2(h + 2g) - m_1 \cdot K_1 - m_2 \cdot K_2 \\ &\quad - A_2/2 + [B + 2(c_2 - c_1) \cdot (h + 2g)]q_1 + A_2q_1 - A_1q_1^2/2 - A_2q_1^2/2 \end{aligned} \quad (15)$$

The port prices disappear in the above expression since the port prices are internalized from the supply chain profit's perspective, i.e. the port revenue and the shipping line's port cost are cancelled out due to their equality. Therefore, we can simply denote the supply chain profit  $\pi(w_1, w_2, q_1)$  as  $\pi(q_1)$ . Let  $q_{1,c}^*$  denote the optimal portcall decision at port 1 under the centralized management model. Define

$$D_2 := \frac{A_2 + B + 2(c_2 - c_1) \cdot (h + 2g)}{A_1 + A_2} \quad (16)$$

The shipping line's optimal portcall decision, the centralized supply chain's optimal profit, and its difference from the decentralized supply chain are given as follows.

**Proposition 3.** Suppose the conditions (12)~(14) are satisfied.

(i) the optimal portcall decision at port 1,  $q_{1,c}^*$ , is given by

$$q_{1,c}^* = \begin{cases} 1 & D_2 > 1 \\ D_2 & 0 \leq D_2 \leq 1 \\ 0 & D_2 < 0 \end{cases} \quad (17)$$

(ii) if  $0 \leq D_2 \leq 1$ , then the centralized supply chain profit under  $q_{1,c}^* = D_2$  is give by

$$\begin{aligned} \pi(q_{1,c}^*) &= (p^h - 2c^h_2)h + p^t \cdot g - c^s_2 - c^t_2 - 2c_2(h + 2g) - m_1 \cdot K_1 - m_2 \cdot K_2 - A_2/2 \\ &\quad + [B + 2(c_2 - c_1) \cdot (h + 2g) + A_2]^2 / [2(A_1 + A_2)] \end{aligned} \quad (18)$$

(iii) if  $0 \leq D_2 \leq 1$ , then the difference in total profits between the centralized supply chain and the decentralized supply chain is given by:

$$[2B + 4(c_2 - c_1) \cdot (h + 2g) + A_2 - A_1]^2 / [18(A_1 + A_2)] \quad (19)$$

Note that the portcall fraction in (17) is different from (11). Since (17) maximizes the total supply chain profit, it is clear that the total profit of the centralized supply chain (under (17)) is not less than the total profit of the decentralized supply chain (under (11)). This intuition is confirmed in Proposition 3(iii).

## 5. Game cost models for two ports and one carrier with multiple services in uncertain demand situations

In reality, customer demands are often subject to uncertainty. A significant number of studies on container shipping have considered uncertain demands, e.g. Dong and Song (2009) and Meng et al. (2012). However, very few studies on port competition have considered demand uncertainty. This section extends the non-cooperative game cost model to situations with uncertain demand volumes. We assume that the hinterland shipments and the transshipment demand are represented by two random variables  $\xi$  and  $\eta$  respectively, s.t.  $E\xi = h$ ,  $E\eta = g$ ,  $\text{Var}(\xi) = \sigma_h^2$ , and  $\text{Var}(\eta) = \sigma_g^2$ . Here,  $\sigma_h$  and  $\sigma_g$  are standard deviations of  $\xi$  and  $\eta$  respectively.

In the uncertain demand situation, the shipping line's expected profit is given by,

$$\pi^l = E \sum_j [(p^h - 2c_j^h - 2w_j) \cdot \xi \cdot q_j + (p^t - 4w_j) \cdot \eta \cdot q_j - c_j^s \cdot q_j - c_j^t \cdot q_j - G_j - H_j] \quad (20)$$

where  $G_j = a_j \cdot [2(\xi + 2\eta) \cdot q_j / K_j]^n$ , and  $H_j = b_j \cdot [2\xi \cdot q_j / R_j]^n$ , for  $j=1,2$ . In the case  $n = 2$ , Eq. (20) can be re-written as

$$\begin{aligned} \pi^l = & (p^h - 2c_2^h - 2w_2) \cdot h + (p^t - 4w_2) \cdot g - c_2^s - c_2^t + (B + 2(w_2 - w_1)(h + 2g))q_1 \\ & - \frac{4a_1(h + 2g)^2 q_1^2}{K_1^2} - \frac{4a_2(h + 2g)^2 (1 - q_1)^2}{K_2^2} - \frac{b_1(2hq_1)^2}{R_1^2} - \frac{b_2(2h(1 - q_1))^2}{R_2^2} \\ & - \frac{4a_1(\sigma_h^2 + 4\sigma_g^2)q_1^2}{K_1^2} - \frac{4a_2(\sigma_h^2 + 4\sigma_g^2)(1 - q_1)^2}{K_2^2} - \frac{4b_1\sigma_h^2 q_1^2}{R_1^2} - \frac{4b_2\sigma_h^2 (1 - q_1)^2}{R_2^2} \end{aligned} \quad (21)$$

Eq. (21) yields the following result by letting  $\partial \pi^l / \partial q_1 = 0$ .

**Lemma 7.** For the given port prices  $w_1$  and  $w_2$  in the uncertain demand situation, the shipping line's optimal portcall decisions are given by

$$q_1^* = \begin{cases} 0 & D_3 < 0 \\ D_3 & 0 \leq D_3 \leq 1; \\ 1 & D_3 > 1 \end{cases}$$

and  $q_2^* = 1 - q_1^*$ ; where

$$D_3 = \frac{B + 2(w_2 - w_1) \cdot (h + 2g) + A_2 + E_2}{A_1 + A_2 + E_1 + E_2}; \quad (22)$$

$$E_1 = 8a_1 \cdot (\sigma_h^2 + 4\sigma_g^2) / K_1^2 + 8b_1 \cdot \sigma_h^2 / R_1^2; \quad (23)$$

$$E_2 = 8a_2 \cdot (\sigma_h^2 + 4\sigma_g^2) / K_2^2 + 8b_2 \cdot \sigma_h^2 / R_2^2. \quad (24)$$

**Proposition 4.** Under the conditions  $0 \leq D_3 \leq 1$ ,  $L_1 \leq w_1 \leq U_1$ , and  $L_2 \leq w_2 \leq U_2$ , the optimal decisions of the shipping line and two ports in uncertain demand situations,  $(w_1^*, w_2^*, \text{ and } q_1^*)$ , are given by (with  $q_2^* = 1 - q_1^*$ ):

$$w_1^* = \frac{A_1 + 2A_2 + B + E_1 + 2E_2}{6(h + 2g)} + \frac{2c_1 + c_2}{3} \quad (25)$$

$$w_2^* = \frac{2A_1 + A_2 - B + 2E_1 + E_2}{6(h + 2g)} + \frac{c_1 + 2c_2}{3} \quad (26)$$

$$q_1^* = \frac{A_1 + 2A_2 + B + 2(c_2 - c_1) \cdot (h + 2g) + E_1 + 2E_2}{3(A_1 + A_2 + E_1 + E_2)} \quad (27)$$

Proposition 4 provides the Nash equilibrium solution to the non-cooperative game in an uncertain demand situation. The conditions in Proposition 4 can be replaced by the following explicit inequalities by substituting  $(w_1, w_2)$  with  $(w_1^*, w_2^*)$ :

$$\frac{-A_2 - E_2}{2(h+2g)} \leq \frac{A_1 - A_2 + B + E_1 - E_2}{6(h+2g)} + \frac{c_2 - c_1}{3} \leq \frac{A_1 + E_1}{2(h+2g)} \quad (28)$$

$$L_1 \leq \frac{A_1 + 2A_2 + B + E_1 + 2E_2}{6(h+2g)} + \frac{2c_1 + c_2}{3} \leq U_1 \quad (29)$$

$$L_2 \leq \frac{2A_1 + A_2 - B + 2E_1 + E_2}{6(h+2g)} + \frac{c_1 + 2c_2}{3} \leq U_2 \quad (30)$$

**Proposition 5.** Under the conditions (12)~(14) and (28)~(30): (i) two ports' optimal prices in uncertain demand situations are greater than those in the deterministic demand situations; (ii) two ports' optimal prices are increasing as the standard deviations of demand uncertainties increase; (iii) the analytical expressions of the optimal profits for two ports and the shipping line can be obtained by inserting (25)~(27) into Eqs. (7) and (21).

Proposition 5 (i)-(ii) can be obtained by comparing (25)~(26) with (9)~(10) respectively, together with the definitions of  $E_1$  and  $E_2$  in relation to the standard deviations  $\sigma_h$  and  $\sigma_g$ . The results indicate that at least one port will definitely benefit from the uncertainty in hinterland shipment and transshipment volume, and such benefit is increasing as the standard deviations of the uncertainties increase. This may be explained as follows: firstly, the fluctuation of demands leads to uneven traffics at ports, which result in more severe peaks and troughs at ports than with deterministic situations. This enables ports to justify higher terminal handling charges. Secondly, since the same amount of total shipment volume (statistically) has to be handled at two ports in total, higher port handling charges will generate more revenue for at least one port. The numerical examples show that both ports can benefit from the uncertainty in customer demands. On the other hand, the shipping line will be worse off in the uncertain demand situations because it has to pay higher port handling charges than in the deterministic demand situations.

It is interesting to contrast our results to the relevant literature in other transport modes. D'Ouille and McDonald (1990) analyzed the effect of uncertain demand on optimal highway capacity and congestion tolls. They found that the optimal capacity under demand uncertainty exceeded that for the certainty case, whereas the optimal toll may be either larger or smaller (depending on the parameters of the problem) than that for the certainty case, which confirmed the numerical results in Kraus (1982). Xiao et al. (2013) examined the effects of demand uncertainty on airport capacity choices. They showed that the optimal airport capacity under demand uncertainty will be larger than the certainty case if demand variation is high or capacity cost is low. The implication is that the uncertain demand could have a significant impact on pricing and capacity decisions.

## 6. A case study

In this section, we will verify some of the analytical results and further explore the managerial insights through a case study with different scenarios. Four groups of experiments are reported. In the first group, we examine the impact of the hinterland volume and the transshipment volume on the optimal decisions (two ports' pricing decisions and the shipping line's portcall decision) and on the players' profits. In the second group, we show the effect of fuel price and the hinterland transportation cost on the optimal decisions and the players'

profits. In the third group, we evaluate the impact of the port congestion cost coefficient and the port handling capacity on the system performance. In the above three groups, the centralized management model and the decentralized non-cooperative model are compared in terms of the portcall decision and the relative supply chain profit. In the fourth group, we illustrate the results of the game model in uncertain demand situations in comparison with the results of the deterministic situations.

### **The base experiment settings**

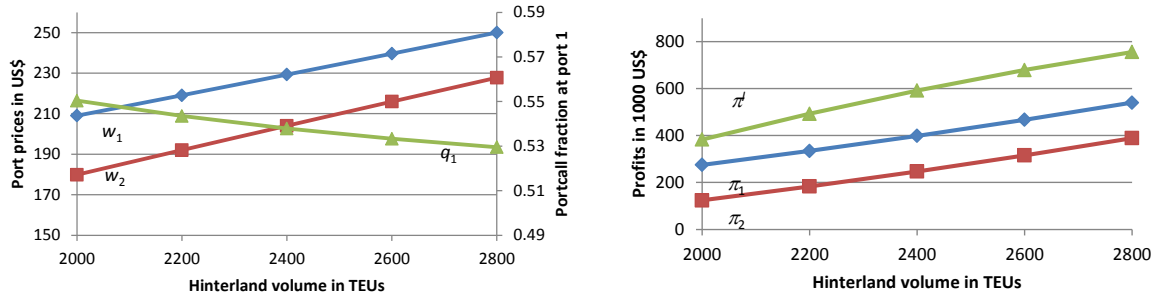
The experiment settings will be based on the case in Section 2: Southampton versus Liverpool. We calibrate the data into two ports-of-call per day for the main services. In the base scenario, we assume ports' daily handling capacity  $K_1 = K_2 = 6000$  TEUs, which is equivalent to an annual capacity  $360 \times 6000 = 2.16$  million TEUs, which is close to Southampton's capacity and also to Liverpool's capacity after the development of Liverpool 2. For every two portcalls, the hinterland shipments  $h = 2000$  TEUs, and the transshipment volume  $g = 300$  TEUs with the projection that two ports would attract slightly more transshipments after the development of Liverpool's new container terminal. That means on average there are 1100 TEUs per port-of-call. The shipment price  $p^h = p^f = 2000$  US\$/TEU is based on the current freight rates in two major shipping routes: Europe-Asia and Trans-Atlantic routes. The deep sea vessel fuel cost at sea per port-of-call at port  $j$  (i.e.  $c_j^s$ ) is defined as the first term in Eq. (1) multiplied by 2; the feeder vessel fuel cost at sea per port-of-call at port  $j$  (i.e.  $c_j^f$ ) is defined as the fourth term in Eq. (1) multiplied by 2, because we assumed two portcalls per day for the main service. The port congestion cost coefficient is set as 500,000 US\$, which represents the cost to the shipping line when the port's utilization reaches its maximum capacity (i.e. extremely long waiting time). This is about 50 days' charter hire for a 10,000 TEU vessel (Ronen 2011). It should be noted that the value of the congestion cost coefficient in this paper is somewhat hypothetical due to the lack of the real data. The ports' unit operating cost ( $c_j$ ) is assumed to be 50 US\$/TEU at both ports. The unit handling capacity investment is assumed to be 30 US\$/TEU, which is based on the Liverpool 2 project, i.e. £300 million investment for 600,000 TEU annual capacity amortized over 25 years ([www.co.uk/projects/liverpool2](http://www.co.uk/projects/liverpool2)). As there is a lack of data in terms of hinterland transport capacity and congestion costs, we neglect them in this case study by assuming infinite capacity or zero congestion cost for the hinterland transport.

Similarly to Section 2, the fuel price  $c^{fuel} = 400$  US\$/tonne, and Southampton and Liverpool have the same unit hinterland transport cost, i.e.  $c_1^h = c_2^h = 300$  US\$/TEU in the base scenario.

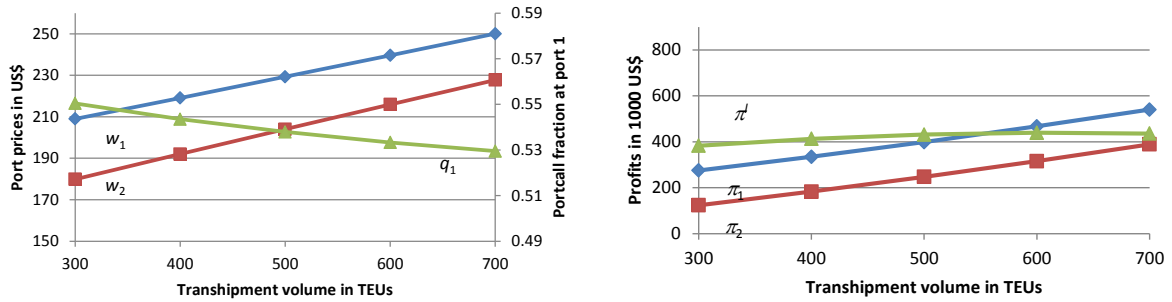
### **6.1 Impact of the hinterland volume and the transshipment volume**

Firstly, we vary the hinterland shipment volume from 2000 TEUs to 2200, 2400, 2600, and 2800 TEUs, but keep other parameters the same as the base scenario. Figure 4 shows the impact of the hinterland shipment volume on the decision variables and the players' profits. Secondly, we vary the transshipment volume from 300 TEUs to 400, 500, 600, and 700 TEUs, but keep other parameters the same as the base scenario. Figure 5 shows the impact of the transshipment volume on the decision variables and the players' profits. In the figures below, the same scale of the vertical axis is used, e.g. the port prices are displayed in the range from \$150 to \$250, the portcall fraction at port 1 is displayed in the range from 0.49 to 0.59, and the players' profits are displayed in the range from 0 to \$900 K.

From Lemma 4, it is easy to calculate that  $\partial w_1^*/\partial h > 0$ ,  $\partial w_2^*/\partial h > 0$ ,  $\partial q_1^*/\partial h < 0$ ,  $\partial w_1^*/\partial g > 0$ ,  $\partial w_2^*/\partial g > 0$  and  $\partial q_1^*/\partial g < 0$ , which indicates that the optimal port prices are increasing whereas the optimal portcall at port 1 is decreasing in both sets of cases, as shown in Figure 4(a) and Figure 5(a). The two ports' profits are increasing in both sets of cases, and the port prices are more sensitive to the transshipment volume. This is in agreement with intuition. However, the shipping line's profit is less intuitive. In Figure 4, the shipping line's profit is increasing, whereas in Figure 5, the shipping line's profit is increasing first and then decreasing. The complicated response of the shipping line's profit may be explained by the fact that, as shown in Figure 5, the port prices are increasing much quicker than in Figure 4 and the transshipment incurs double handling charges at ports compared to the hinterland shipment. This implies that the shipping line's profit from an additional transshipment may be cancelled out by the increasing port charges at a certain transshipment level. For example, when the transshipment volume  $g$  increases from 300 TEUs to 700 TEUs in Figure 5, the port prices  $w_1^*$  and  $w_2^*$  are increasing from \$209 and \$180 to \$250 and \$228, respectively. Such significant increases in port prices create a considerable impact on the shipping line's profit and diminishes the profit generated from the additional transshipment volume.



(a) Impact on decision variables (b) Impact on profits  
Figure 4. Impact of the hinterland shipment volume



(a) Impact on decision variables (b) Impact on profits  
Figure 5. Impact of the transshipment volume

Table 1 compares the results of the decentralized supply chain and the centralized supply chain for the cases in Figures 4 and 5, where  $q_{1,d}^*$  and  $\pi_d^*$  represent the optimal portcall fraction at port 1 and the supply chain profit in the decentralized non-cooperative model, and  $q_{1,c}^*$  and  $\pi_c^*$  represent the optimal portcall fraction at port 1 and the supply chain profit in the centralized management model. Table 1 verifies the result in Proposition 3, which provides the difference in the supply chain profits between the centralized and decentralized models. It can be observed that in the centralized management model, port 1 would have a larger share of portcalls than the decentralized non-cooperative model. However, the difference in the

supply chain profits is decreasing as either the hinterland shipment or the transshipment volume increases. Moreover, the difference appears to be minor (around a few thousand US\$), which indicates that the full integration of the transport chain does not offer significant benefit from the supply chain profit perspective in our case study.

Table 1. Decentralized, non-cooperative model versus centralized model with varying shipment volume

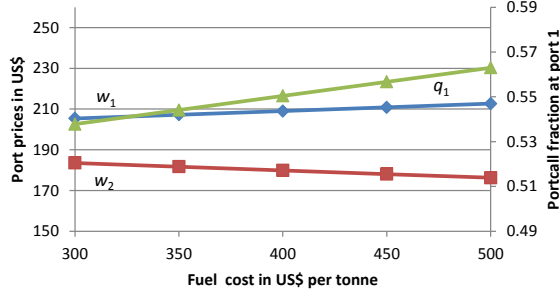
Varying hinterland shipment volume				Varying transshipment volume			
$h$ (TEU)	$q_{1,d}^*$	$q_{1,c}^*$	$\pi_c^* - \pi_d^*$	$g$ (TEU)	$q_{1,d}^*$	$q_{1,c}^*$	$\pi_c^* - \pi_d^*$
2000	0.550	0.651	\$7636	300	0.550	0.651	\$7636
2200	0.544	0.630	\$6584	400	0.544	0.630	\$6584
2400	0.538	0.614	\$5736	500	0.538	0.614	\$5736
2600	0.533	0.600	\$5041	600	0.533	0.600	\$5041
2800	0.530	0.588	\$4466	700	0.530	0.588	\$4466

## 6.2 Effect of the fuel price and the hinterland transportation cost

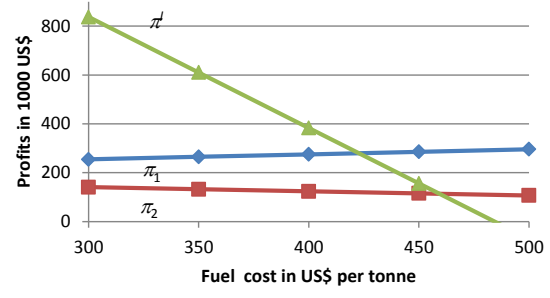
In this sub-section, we first vary the fuel price from 300 US\$/tonne to 350, 400, 450, and 500 US\$/tonne, but keep other parameters the same as the base scenario. Figure 6 shows the impact of the fuel price on the decision variables and the players' profits. Secondly, we vary the unit hinterland transportation cost at port 1 (i.e.  $c_1^h$ ) from \$300 to \$315, \$330, \$345, and \$360, but keep other parameters the same as the base scenario. Figure 7 shows the impact of the hinterland transportation cost on the decision variables and the players' profits.

From Lemma 5, and the definitions of  $c_1^s$ ,  $c_2^s$ ,  $c_1^t$ ,  $c_2^t$ , we can derive that  $\partial w_1^* / \partial c^{fuel} > 0$ ,  $\partial w_2^* / \partial c^{fuel} < 0$ ,  $\partial q_1^* / \partial c^{fuel} > 0$  in our case study. This is verified by Figure 6(a). In addition, Figure 6(b) shows that as the fuel price increases, the shipping line's profit is decreasing rapidly. In fact, the shipping line's profit becomes negative when fuel price reaches \$500 per tonne in our scenario. In practice, the shipping line may use the bunker adjustment factor (fuel surcharge) to pass the costs to shippers. We can also observe that port 1's profit is increasing due to the increase of port price and the portcall fraction, whereas port 2's profit is decreasing due to the decrease of port price and the portcall fraction at port 2.

Regarding the impact of hinterland transportation cost, Lemma 5 yields for  $\partial w_1^* / \partial c_1^h < 0$ ,  $\partial w_2^* / \partial c_1^h > 0$ ,  $\partial q_1^* / \partial c_1^h < 0$ , which are verified in Figure 7(a). Note that increasing  $c_1^h$  represents the scenarios where port 2 has a close proximity to the hinterland market. This enables port 2 to increase its competitiveness, which is reflected by the increasing port price at port 2, decreasing port price at port 1, and decreasing portcall fraction at port 1 in Figure 7(a). In fact, the optimal portcall fraction at port 2 becomes greater than that at port 1 when  $c_1^h$  approaches to \$360 (note that  $c_2^h = \$300$ ). The players' profits in Figure 7(b) are the combined effect of the port prices and the portcall split over the two ports. Generally, as  $c_1^h$  increases, the shipping line's profit is decreasing, whereas port 2's profit is increasing due to gaining more market share.

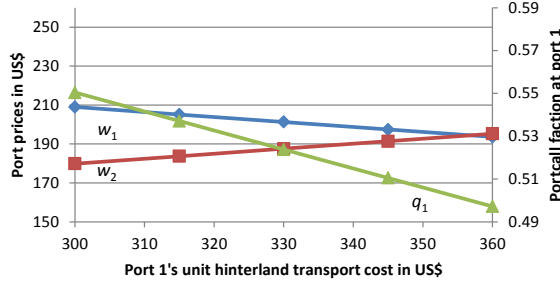


(a) Impact on decision variables

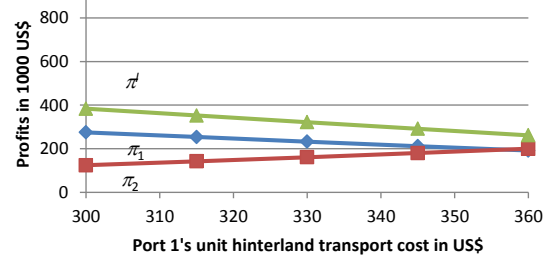


(b) Impact on profits

Figure 6. Impact of the fuel cost



(a) Impact on decision variables



(b) Impact on profits

Figure 7. Impact of the unit hinterland transportation cost at port 1

Table 2 compares the results of the decentralized supply chain and the centralized supply chain for the cases in Figures 6 and 7. Similar to Table 1, it confirms that: (i) the centralized model achieves a higher supply chain profit than the decentralized non-cooperative model in all cases; (ii) the difference in the supply chain profits appears to be minor in our cases. However, different to the results presented in Table 1, port 1 could have a smaller share of portcalls in the centralized model than in the decentralized model when the unit hinterland transportation cost at port 1 (i.e.  $c_1^h$ ) increases to a certain point (e.g. \$360), which represents the situation that port 1 become less competitive than port 2. In addition, it shows that the profit difference is increasing with respect to the fuel cost, and has a U-shape with respect to the unit hinterland transportation cost of port 1 (after experimenting with larger values of  $c_1^h$ ).

Table 2. Decentralized, non-cooperative model versus centralized model with varying fuel cost and port 1's hinterland transportation cost

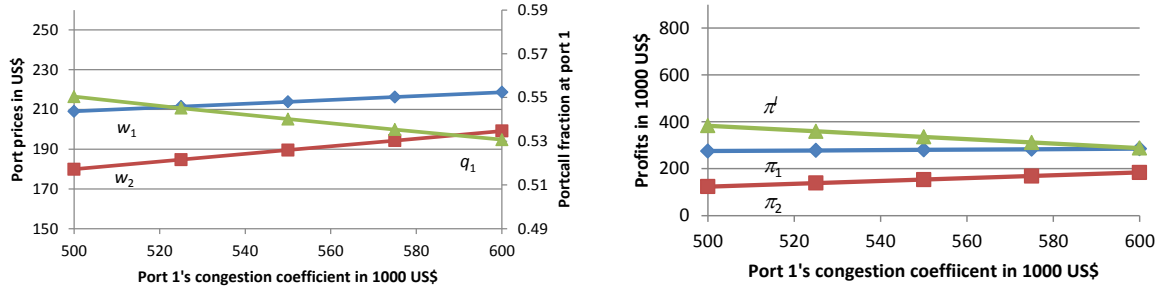
Varying fuel cost				Varying port 1's hinterland transport cost			
$c^{fuel}$	$q_{1,d}^*$	$q_{1,c}^*$	$\pi_c^* - \pi_d^*$	$c_1^h$	$q_{1,d}^*$	$q_{1,c}^*$	$\pi_c^* - \pi_d^*$
300	0.538	0.613	\$4295	300	0.550	0.651	\$7636
350	0.544	0.632	\$5847	315	0.537	0.611	\$4136
400	0.550	0.651	\$7636	330	0.524	0.571	\$1700
450	0.557	0.670	\$9665	345	0.511	0.531	\$330
500	0.563	0.689	\$11932	360	0.497	0.492	\$24

### 6.3 Impact of the port congestion cost coefficient and the port handling capacity

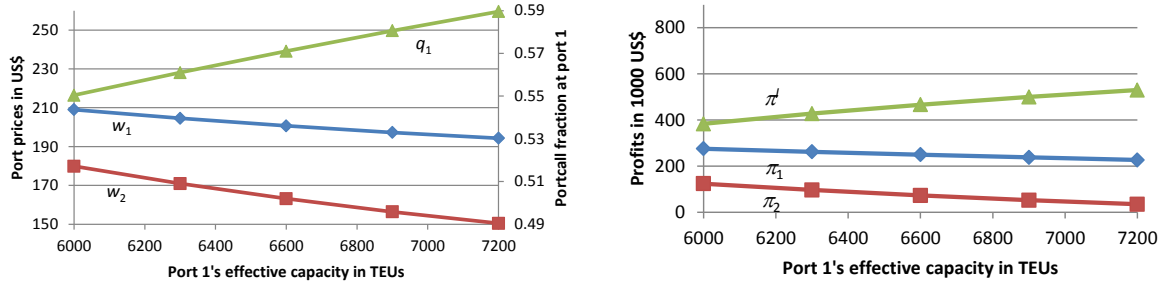


In this sub-section, we first vary port 1's congestion cost coefficient (i.e.  $a_1$ ) in 5% intervals, i.e. from \$500000, to \$525000, \$550000, \$575000, and \$600000, but keep other parameters the same as the base scenario. Figure 8 shows the impact of port 1's congestion cost on the decision variables and the players' profits. Secondly, we vary the daily handling capacity (i.e.  $K_1$ ) at port 1 using a 5% interval, i.e. from 6000 TEUs to 6300, 6600, 6900, and 7200 TEUs, but keep other parameters the same as the base scenario. Figure 9 shows the impact of handling capacity at port 1 on the decision variables and the players' profits.

Figure 8(a) and Figure 9(a) verify the results in Lemma 6, e.g. both ports' prices are increasing in  $a_1$  and decreasing in  $K_1$  and port 2's price is more sensitive to  $a_1$  and  $K_1$ . The shipping line's portcall fraction at port 1 is decreasing in  $a_1$  and increasing in  $K_1$ , which is intuitively true. Moreover, from Figures 8(b) and 9(b), the shipping line's profit is decreasing in  $a_1$  and increasing in  $K_1$ .



(a) Impact on decision variables (b) Impact on profits  
Figure 8. Impact of the congestion cost coefficient at port 1



(a) Impact on decision variables (b) Impact on profits  
Figure 9. Impact of port 1's handling capacity

Table 3 compares the results of the decentralized supply chain and the centralized supply chain for the cases in Figures 8 and 9. Similar results to Table 1 can be observed. In addition, Table 3 illustrates that the profit difference is decreasing in port 1's congestion cost coefficient, and increasing in port 1's handling capacity.

Table 3. Decentralized, non-cooperative model versus centralized model with varying port 1's congestion cost and handling capacity

Varying port 1's congestion cost				Varying port 1's handling capacity			
$a_1$	$q_{1,d}^*$	$q_{1,c}^*$	$\pi_c^* - \pi_d^*$	$K_1$	$q_{1,d}^*$	$q_{1,c}^*$	$\pi_c^* - \pi_d^*$
500000	0.550	0.651	\$7636	6000	0.550	0.651	\$7636
525000	0.545	0.635	\$6270	6300	0.561	0.683	\$10659
550000	0.540	0.620	\$5067	6600	0.571	0.713	\$13848

575000	0.535	0.606	\$4018	6900	0.581	0.742	\$17121
600000	0.531	0.592	\$3111	7200	0.590	0.769	\$20417

#### 6.4 Comparison of uncertain demands and deterministic demands

We take the scenarios in Sections 6.1 and 6.2 as examples to compare the results under uncertain demands with deterministic demands, in which the coefficient of variation, CoV, takes two levels, 0.2 and 0.4 respectively. The results are shown in Table 4, where the first column provides the combinations of hinterland demands and transshipment demands in TEUs; the second and third columns provide the profit differences in US\$ between uncertain demand situations and deterministic situations for port 1 and port 2 respectively; the fourth column shows the profit differences in US\$ for the shipping line; and the fifth column shows the total supply chain profit differences in US\$.

Table 4. Comparison of deterministic and uncertain demand situations

	$\pi_{1,unc}^* - \pi_{1,det}^*$	$\pi_{2,unc}^* - \pi_{2,det}^*$	$\pi_{unc}^* - \pi_{det}^*$	$\pi_{unc}^* - \pi_{det}^*$
CoV = 0.2				
$(h, g) = (2000, 300)$	44040	17082	-66836	-5713
$(h, g) = (2200, 300)$	53416	20339	-80310	-6556
$(h, g) = (2400, 300)$	63675	23873	-95047	-7500
$(h, g) = (2600, 300)$	74817	27687	-111048	-8544
$(h, g) = (2800, 300)$	86844	31786	-128315	-9685
$(h, g) = (2000, 300)$	44040	17082	-66836	-5713
$(h, g) = (2000, 400)$	44140	16847	-66799	-5811
$(h, g) = (2000, 500)$	44210	16642	-66912	-6060
$(h, g) = (2000, 600)$	44262	16462	-67163	-6439
$(h, g) = (2000, 700)$	44300	16304	-67540	-6936
CoV = 0.4				
$(h, g) = (2000, 300)$	176551	66143	-264813	-22119
$(h, g) = (2200, 300)$	214023	78961	-318530	-25546
$(h, g) = (2400, 300)$	255029	92911	-377315	-29375
$(h, g) = (2600, 300)$	299573	108002	-441172	-33597
$(h, g) = (2800, 300)$	347659	124242	-510108	-38206
$(h, g) = (2000, 300)$	176551	66143	-264813	-22119
$(h, g) = (2000, 400)$	176824	65622	-265194	-22748
$(h, g) = (2000, 500)$	177026	65129	-266049	-23895
$(h, g) = (2000, 600)$	177178	64670	-267359	-25512
$(h, g) = (2000, 700)$	177294	64248	-269108	-27566

It can be seen from Table 4 that both ports gain benefits from the demand uncertainty in hinterland shipments and transshipment volumes, and such benefit is increasing up to 3-4 times when the coefficient of variation (CoV) increases from 0.2 to 0.4; on the other hand, the shipping line's profit and the total supply chain profit are both decreasing as the CoV increases. In addition, under a fixed degree of demand uncertainty (i.e. fixed CoV), the benefits that both ports gained from uncertain demands are increasing in  $h$  (i.e. the average amount of the hinterland shipments); however, the increase in  $g$  (i.e. the average transshipment volume) has a mixed impact on the two ports; the difference in the total supply chain profit

between the uncertain demand situation and the deterministic situation is increasing as either  $h$  or  $g$  increases.

## 7. Extension of the non-cooperative game model to three-port case

In this section, we discuss the extension of the non-cooperative game model to a single shipping line and three-port case. It is assumed that the customer demands are deterministic.

The shipping line's profit function is given by,

$$\pi^l = \sum_j [(p^h - 2c^h_j - 2w_j) \cdot h \cdot q_j + (p^t - 4w_j) \cdot g \cdot q_j - c^s_j \cdot q_j - c^t_j \cdot q_j - G_j - H_j] \quad (31)$$

where  $G_j = 4a_j \cdot [(h + 2g) \cdot q_j / K_j]^2$ ,  $H_j = 4b_j \cdot [h \cdot q_j / R_j]^2$ , for  $j=1,2,3$ , s.t.  $0 \leq q_1, q_2, q_3 \leq 1$ ; and  $q_3 = 1 - q_1 - q_2$ .

The ports' profit functions (for  $j=1,2,3$ ) are given by,

$$\pi_j = (w_j - c_j) \cdot F_j - m_j \cdot K_j = (w_j - c_j) \cdot 2(h + 2g) \cdot q_j - m_j \cdot K_j \quad (32)$$

To simplify the narrative, let

$$A_j := 8a_j \cdot (h + 2g)^2 / K_j^2 + 8b_j \cdot h^2 / R_j^2, \text{ for } j=1,2,3;$$

$$B_1 := 2(c^h_3 - c^h_1) \cdot h + c^s_3 - c^s_1 + c^t_3 - c^t_1;$$

$$B_2 := 2(c^h_3 - c^h_2) \cdot h + c^s_3 - c^s_2 + c^t_3 - c^t_2.$$

Then,  $\partial \pi^l / \partial q_1$  and  $\partial \pi^l / \partial q_2$  can derived as

$$\partial \pi^l / \partial q_1 = -(A_1 + A_3)q_1 - A_3q_2 + A_3 + B_1 + 2(w_3 - w_1) \cdot (h + 2g)$$

$$\partial \pi^l / \partial q_2 = -A_3q_1 - (A_2 + A_3)q_2 + A_3 + B_2 + 2(w_3 - w_2) \cdot (h + 2g)$$

Let the first partial derivatives be zero, i.e.  $\partial \pi^l / \partial q_1 = \partial \pi^l / \partial q_2 = 0$ , we have,

$$q_1 = [-A_3B_2 + A_3B_1 + A_2A_3 + A_2B_1 - 2A_3(w_1 - w_2) \cdot (h + 2g) + 2A_2(w_3 - w_1) \cdot (h + 2g)] / (A_2A_1 + A_2A_3 + A_1A_3) \quad (33)$$

$$q_2 = [A_3B_2 - A_3B_1 + A_1A_3 + A_1B_2 + 2A_3(w_1 - w_2) \cdot (h + 2g) + 2A_1(w_3 - w_2) \cdot (h + 2g)] / (A_2A_1 + A_2A_3 + A_1A_3) \quad (34)$$

$$q_3 = [A_2A_1 - A_2B_1 - A_1B_2 - 2A_2(w_3 - w_1) \cdot (h + 2g) - 2A_1(w_3 - w_2) \cdot (h + 2g)] / (A_2A_1 + A_2A_3 + A_1A_3) \quad (35)$$

Insert the expressions ( $q_1, q_2, q_3$ ) into the three ports' profit function respectively. Let their first derivatives with respect to the corresponding port price  $w_j$  be zero. We will obtain three linear equations with three unknown variables ( $w_1, w_2, w_3$ ):

$$2[-2(A_2 + A_3)w_1 + A_3w_2 + A_2w_3 + c_1 \cdot (A_3 + A_2)](h + 2g) - A_3B_2 + A_3B_1 + A_2A_3 + A_2B_1 = 0;$$

$$2[A_3w_1 - 2(A_3 + A_1)w_2 + A_1w_3 + c_2 \cdot (A_3 + A_2)](h + 2g) + A_3B_2 - A_3B_1 + A_1A_3 + A_1B_2 = 0;$$

$$2[A_2w_1 + A_1w_2 - 2(A_1 + A_2)w_3 + c_3 \cdot (A_1 + A_2)](h + 2g) + A_2A_1 - A_2B_1 - A_1B_2 = 0.$$

Solving the above three linear equations, we can obtain the closed-form of the optimal port pricing decisions ( $w_1^*, w_2^*, w_3^*$ ). Substitute ( $w_1^*, w_2^*, w_3^*$ ) into Eqs. (33)~(35), we then obtain the optimal portcall decisions ( $q_1^*, q_2^*, q_3^*$ ). However, it is tedious to display the optimal solution and therefore we omit it. Nevertheless, the impact of the system parameters on the optimal decisions and the optimal profits of each stakeholder can be analyzed in a similar manner to the two-port case.

## 8. Conclusions

This paper considers a novel port competition problem involving both hinterland shipments and transshipment cargoes, analyzed from the transport chain's cost perspective and taking

into account port handling charges, deep sea transport cost, hinterland transport cost, and feeder service cost. A static cost model is first presented for two competitive ports with specific services to evaluate their relative cost in the transport chain. The case study of Southampton and Liverpool ports illustrates that either of the two competitive ports can be more cost efficient under certain conditions, or by appropriately adjusting their terminal handling charges. For example, higher planned vessel sailing speeds and higher fuel cost would be in favour of selecting Southampton; whereas higher hinterland shipment and higher transshipment volumes would be in favour of selecting Liverpool. In particular, the port handling charges at the two ports have a significant impact on their relative cost profiles. This indicates that it is more likely that each of the competitive ports may attract a fraction of the shipping services, and demonstrates the necessity of modelling port competition within a game framework in the context of the overall transport chain.

We then presented a non-cooperative game model for two competitive ports and one ocean carrier with multiple shipping services concerning both ports' pricing decisions and the ocean carrier's port-of-call decisions. A closed-form of the optimal solution is derived. Revealing managerial insights are established and verified in the case study of Southampton and Liverpool ports, e.g. (i) it is not guaranteed that ports' prices will be increasing as the hinterland shipments or transshipment volume increases. However, as the hinterland shipment or transshipment volume reaches a certain threshold level, the two ports' handling prices are indeed increasing; (ii) when the ocean carrier attaches more weight to the congestion cost on either port, both ports will increase their port prices but to different scales; (iii) when either port's handling capacity increases, both ports' prices are decreasing which leads to the decrease of both ports' profits but to different scales; (iv) the centralized management model achieves a higher supply chain profit than the decentralized non-cooperative model. However, the difference in the supply chain profits in both models is rather small in our cases. Southampton would have a larger share of portcalls in the centralized model than in the decentralized model in most scenarios; (v) in the presence of uncertainty in hinterland shipments and transshipment volumes, it is shown that both ports will increase their port handling charges. Numerical examples show that both ports can benefit from such uncertainty in comparison with deterministic demand situations; and the benefit is increasing as the degree of uncertainty increases. On the other hand, the shipping line will be worse off in uncertain demand situations. The above managerial insights can serve as useful information to the port operators and the shipping line when formulating their port pricing strategies and port-of-call decisions, and pursuing a policy of supply chain integration.

Further research could be undertaken in the following directions. First, multiple-port competition is more realistic or applicable than two-port competition. Although we provided an extension of the non-cooperative game model to the three-port competition case, more in-depth research is required. Second, we focused on a single shipping line (or a shipping alliance), which neglected the competition between shipping lines. It would be instructive to extend the model to multiple shipping line cases, e.g. Bae et al. (2013) considered multiple identical shipping lines. Third, as port capacity choice is an important issue and interwoven with pricing decisions, it would be of potential value to investigate the joint capacity investment and price decision problem. Fourth, this paper focused on ports' price decisions and shipping line's portcall decisions. It would be desirable to include shippers' decision in the model. In addition, the environmental aspect of the performance measures such as emissions could be included in a revised model.

## Acknowledgements

The authors are grateful to three anonymous reviewers for their valuable and constructive comments. The research was partly supported by EC FP7 (Grant No. PIRSES-GA-2013-612546) and by China NSFC (Grant No. 71772071).

## References

1. Anderson, C.M., Park, Y.A., Chang, Y.T., Yang, C.H., Lee, T.W. & Luo, M.F. (2008), A game-theoretic analysis of competition among container port hubs: the case of Busan and Shanghai, *Maritime Policy & Management*, 35(1), 5-26,
2. Bae, M.J., Chew, E.K., Lee, L.H. & Zhang, A. (2013). Container transshipment and port competition, *Maritime Policy & Management*, 40(5), 479-494.
3. Barnard, B. (2014), Europe's ports brace for intra-terminal rivalries and overcapacity, *Journal of Commerce*, Friday 7th November, 2014.
4. Brouer, B.D., Alvarez, J.F., Plum, C.E.M., Pisinger, D. and Sigurd, M.M. (2014). A base integer programming model and benchmark suite for liner-shipping network design. *Transportation Science*, 48 (2), 281–312.
5. Cariou, P., (2011), Is slow steaming a sustainable means of reducing CO2 emissions from container shipping? *Transportation Research Part D*, 16, 260–264.
6. Cullinane, K., Teng, Y. & Wang, T.F. (2005), Port competition between Shanghai and Ningbo, *Maritime Policy & Management*, 32(4), 331-346.
7. De Borger, B., Proost, S. and Van Dender, K. (2008). Private port pricing and public investment in port and hinterland capacity, *Journal of Transport Economics and Policy*, 42, 527–561.
8. De Borger, B., and Van Dender, K. (2006). Prices, capacities and service levels in a congestible Bertrand duopoly. *Journal of Urban Economics*, 60, 264–283.
9. Dong, J.X. and Song, D.P., (2009). Simulation-based optimization for container fleet sizing and empty repositioning in liner shipping systems. *Transportation Research Part E*, 45(6), 860-877.
10. D'Ouille, E., McDonald, J.F., (1990). Effects of demand uncertainty on optimal capacity and congestion tolls for urban highways. *Journal of Urban Economics*, 28 (1), 63–70.
11. Ishii, M., Lee, P.T.W., Tezuka, K. and Chang, Y.T. (2013), A game theoretical analysis of port competition, *Transportation Research Part E*, 49(1), 92-106.
12. Kraus, M. (1982). Highway pricing and capacity choice under uncertainty, *Journal of Urban Economics*, 12, 122-128.
13. Lee, P.T. and Flynn, M. (2011). Charting a new paradigm of container port development policy: the Asian doctrine, *Transport Reviews*, 31(6), 791-806.
14. Li, J.B. and Oh, Y.S. (2010), A research on competition and cooperation between Shanghai port and Ningbo-Zhoushan port, *The Asian Journal of Shipping and Logistics*, 26(1), 67–91.
15. Liu, Z., Meng, Q., Wang, S. and Sun, Z. (2014), Global intermodal liner shipping network design, *Transportation Research Part E*, 61, 28–39.
16. Luo, M. and Grigalunas, T. (2003). A spatial-economic multimodal transportation simulation model for US coastal container ports. *Maritime Economics and Logistics*, 5(2), 158–178.
17. Luo, M., Liu, L. and Gao, F. (2012), Post-entry container port capacity expansion, *Transportation Research Part B*, 46, 120–138.

18. Meng, Q., Wang, S., Andersson, H. and Thun, K. (2014). Containership routing and scheduling in liner shipping: Overview and future research directions. *Transportation Science*, 48(2), 265–280.
19. Meng, Q., Wang, T. and Wang, S. (2012). Short-term liner ship fleet planning with container transshipment and uncertain container shipment demand, *European Journal of Operational Research*, 223(1), 96-105.
20. Notteboom, T., Parola, F., Satta, G. (2014). Deliverable 1.1: State of the European Port System – market trends and structure update Partim transshipment volumes, the 7th Framework Programme: PORTOPIA.
21. Notteboom, T. and Yap, W.Y. (2012). Port competition and competitiveness. In: Tally, W. (Ed.), *The Blackwell Companion to Maritime Economics*, Wiley-Blackwell, pp.549–570.
22. Pocuca, M. (2006), Methodology of day-to-day ship costs assessment, *Promet- Traffic & Transportation*, 18(5), 337-345.
23. Porter, J. (2013), Tilbury confident as London box ports battle for market share, *Lloyds List*, Monday 09 September 2013.
24. Ronen, D., (2011), The effect of oil price on containership speed and fleet size. *Journal of Operational Research Society*, 62, 211–216.
25. Saeed, N. and Larsen, O.I. (2010). An application of cooperative game among container terminals of one port. *European Journal of Operational Research*, 203(2), 392–403.
26. Song, D.W. (2002). Regional container port competition and co-operation: the case of Hong Kong and South China. *Journal of Transport Geography*, 10(2), 99-110.
27. Song, D.P. and Dong, J.X. (2013). Long-haul liner service route design with ship deployment and empty container repositioning. *Transportation Research Part B*, 55, 188-211.
28. Tavasszy, L. A., Minderhoud, M., Perrin, J.-F., & Notteboom, T. (2011). A strategic network choice model for global container flows: specification, estimation and application. *Journal of Transport Geography*, 19(6), 1163-1172.
29. Tongzon, J. and Heng, W. (2005). Port privatization, efficiency and competitiveness: Some empirical evidence from container ports (terminals). *Transportation Research Part A*, 39(5), 405–424.
30. Veldman, S.J. and Buckmann, E.H. (2003), A model on container port competition: an application for the west European container hub-ports, *Maritime Economics & Logistics*, 5, 3–22.
31. Wiegman B.W., van der Hoest A., Notteboom T. (2008), Port and terminal selection by deep-sea container operators. *Maritime Policy and Management*, 35(6), 517-534.
32. Xiao Y., Fu X. and Zhang A. (2013). Demand uncertainty and airport capacity choice, *Transportation Research Part B*, 57, 91–104
33. Yap, W. Y., and J. S. L. Lam. (2006). Competition dynamics between container ports in East Asia. *Transportation Research Part A*, 40, 35–51.
34. Yeo, G.T., Roe, M. and Dinwoodie, J. (2008). Evaluating the competitiveness of container ports in Korea and China, *Transportation Research Part A*, 42, 910-921.
35. Zhuang W., Fu X. and Luo M. (2014). A game theory analysis of port specialization – implications to the Chinese port industry, *Maritime Policy & Management*, 41(3), 268–287.

## Appendix

### Proof of Lemma 2.

For fixed port prices ( $w_1, w_2$ ), take the first partial derivative of Eq. (8) with respect to  $q_1$ , we have,

$$\begin{aligned} \partial \pi / \partial q_1 = & [(p^h - 2c^h_1 - 2w_1) \cdot h + (p^t - 4w_1) \cdot g - c^s_1 - c^t_1] \\ & - [(p^h - 2c^h_2 - 2w_2) \cdot h + (p^t - 4w_2) \cdot g - c^s_2 - c^t_2] \\ & - \frac{2^n n a_1 (h+2g)^n q_1^{n-1}}{K_1^n} + \frac{2^n n a_2 (h+2g)^n (1-q_1)^{n-1}}{K_2^n} - \frac{2^n n b_1 h^n q_1^{n-1}}{R_1^n} + \frac{2^n n b_2 h^n (1-q_1)^{n-1}}{R_2^n} \end{aligned}$$

It follows (note that  $n \geq 1$ ):  $\partial^2 \pi / \partial^2 q_1 \leq 0$ . Thus, Eq. (8) is concave with respect to  $q_1$  in the interval  $[0, 1]$ . This completes the proof of Lemma 2.

### Proof of Proposition 1.

It is easy to observe that the condition  $D_1 < 0$  is equivalent to  $w_2 - w_1 < (-A_2 - B)/(2h + 4g)$ ;  $D_1 > 1$  is equivalent to  $w_2 - w_1 > (A_1 - B)/(2h + 4g)$ ; and  $0 \leq D_1 \leq 1$  is equivalent to  $(-A_2 - B)/(2h + 4g) \leq w_2 - w_1 \leq (A_1 - B)/(2h + 4g)$ .

Under the condition  $D_1 < 0$ , each port's profit function becomes linear to its price. Hence, both ports would choose the port prices as high as possible subject to  $D_1 < 0$  to maximize their profits, which leads to Proposition 1(i).

Under the condition  $D_1 > 1$ , similarly, each port's profit function is also linear to its price. Thus, both ports would choose the port prices as high as possible subject to  $D_1 > 1$  to maximize their profits, which leads to Proposition 1(ii).

Under the condition  $0 \leq D_1 \leq 1$ , we have,

$$\begin{aligned} \pi_1 &= \frac{2(w_1 - c_1)(h+2g)[A_2 + B + 2(w_2 - w_1)(h+2g)]}{A_1 + A_2} - m_1 K_1 \\ \pi_2 &= \frac{2(w_2 - c_2)(h+2g)[A_1 - B - 2(w_2 - w_1)(h+2g)]}{A_1 + A_2} - m_2 K_2 \end{aligned}$$

Let the first partial derivatives  $\partial \pi_1 / \partial w_1 = \partial \pi_2 / \partial w_2 = 0$ . We have,

$$w_1^* = \frac{(A_1 + 2A_2) + B}{6(h+2g)} + \frac{2c_1 + c_2}{3}, \text{ and } w_2^* = \frac{(2A_1 + A_2) - B}{6(h+2g)} + \frac{c_1 + 2c_2}{3}$$

Substituting  $w_i$  with  $w_i^*$  in the conditions  $0 \leq D_1 \leq 1$ ,  $L_1 \leq w_1 \leq U_1$ , and  $L_2 \leq w_2 \leq U_2$ , these conditions can be given in explicit forms as follows:

$$\begin{aligned} -A_2 &\leq \frac{(A_1 - A_2) + B + 2(c_2 - c_1)(h+2g)}{3(h+2g)} \leq A_1 \\ L_1 &\leq \frac{(A_1 + 2A_2) + B}{6(h+2g)} + \frac{2c_1 + c_2}{3} \leq U_1 \\ L_2 &\leq \frac{(2A_1 + A_2) - B}{6(h+2g)} + \frac{c_1 + 2c_2}{3} \leq U_2 \end{aligned}$$

The shipping line's optimal price can be derived in closed-form by Lemma 3. This completes the proof of Proposition 1.

### Proof of Proposition 3.

Take the first derivative of (15), we have,

$$d\pi(q_1)/dq_1 = B + 2(c_2 - c_1) \cdot (h+2g) + A_2 - (A_1 + A_2)q_1$$

It is clear that  $D_2$  is the solution to  $d\pi(q_1)/dq_1 = 0$ . Note that any  $q_1 \in [0, 1]$ . It yields Proposition 3(i). Moreover, the centralized supply chain profit under  $q_{1,c}^* = D_2$  is give in Proposition 3(ii). Finally, Proposition 3(iii) can be derived by some algebraic manipulation. Note that,

$$\begin{aligned}\pi(q_{1,c}^*) &= (p^h - 2c^h_2)h + p^t g - c^s_2 - c^t_2 - c_2 \cdot 2(h + 2g) - m_1 \cdot K_1 - m_2 \cdot K_2 - A_2 / 2 \\ &\quad + [B + (c_2 - c_1) \cdot 2(h + 2g) + A_2] \cdot q_{1,c}^* - A_1 q_{1,c}^{*2} / 2 - A_2 q_{1,c}^{*2} / 2 \\ \pi(q_{1,c}^*) &= (p^h - 2c^h_2)h + p^t \cdot g - c^s_2 - c^t_2 - 2c_2(h + 2g) - m_1 \cdot K_1 - m_2 \cdot K_2 - A_2 / 2 \\ &\quad + [B + 2(-c_1 + c_2) \cdot (h + 2g)] q_{1,c}^* + A_2 q_{1,c}^* - A_1 q_{1,c}^{*2} / 2 - A_2 q_{1,c}^{*2} / 2\end{aligned}$$

It follows,

$$\begin{aligned}\pi(q_{1,c}^*) - \pi(q_1^*) &= [B + 2(c_2 - c_1) \cdot (h + 2g) + A_2] (q_{1,c}^* - q_1^*) - (A_1 + A_2)(q_{1,c}^{*2} - q_1^{*2}) / 2 \\ &= (q_{1,c}^* - q_1^*) [2B + 4(c_2 - c_1) \cdot (h + 2g) + 2A_2 - (A_1 + A_2)(q_{1,c}^* + q_1^*)] / 2 \\ &= [2B + 4(c_2 - c_1) \cdot (h + 2g) + A_2 - A_1]^2 / [18(A_1 + A_2)^2]\end{aligned}$$

where  $q_{1,c}^*$  is given in (16) for the centralized supply chain's optimal portcall decision; and  $q_1^*$  is given in (11) for the decentralized supply chain's optimal portcall decision. This completes the proof.

#### Proof of Proposition 4:

It is easy to see that the condition  $0 \leq D_3 \leq 1$  is equivalent to:

$$\frac{-A_2 - E_2 - B}{2(h + 2g)} \leq w_2 - w_1 \leq \frac{A_1 + E_1 - B}{2(h + 2g)}$$

Note that the ports' expected profits (for  $j=1,2$ ) under the port price  $w_j$  are given by,

$$\begin{aligned}\pi_j &= E [(w_j - c_j) \cdot 2(\xi + 2\eta) \cdot q_j - m_j \cdot K_j] \\ &= (w_j - c_j) \cdot 2(h + 2g) \cdot q_j - m_j \cdot K_j\end{aligned}$$

From Lemma 7, it follows,

$$\begin{aligned}\pi_1 &= 2(w_1 - c_1) \cdot (h + 2g) \cdot \frac{B + 2(w_2 - w_1)(h + 2g) + A_2 + E_2}{A_1 + A_2 + E_1 + E_2} - m_1 K_1 \\ \pi_2 &= 2(w_2 - c_2) \cdot (h + 2g) \cdot \frac{-B - 2(w_2 - w_1)(h + 2g) + A_1 + E_1}{A_1 + A_2 + E_1 + E_2} - m_2 K_2\end{aligned}$$

Let the first partial derivatives  $\partial\pi_1/\partial w_1 = \partial\pi_2/\partial w_2 = 0$ , together with Lemma 7, we can obtain the results in Proposition 4. This completes the proof.